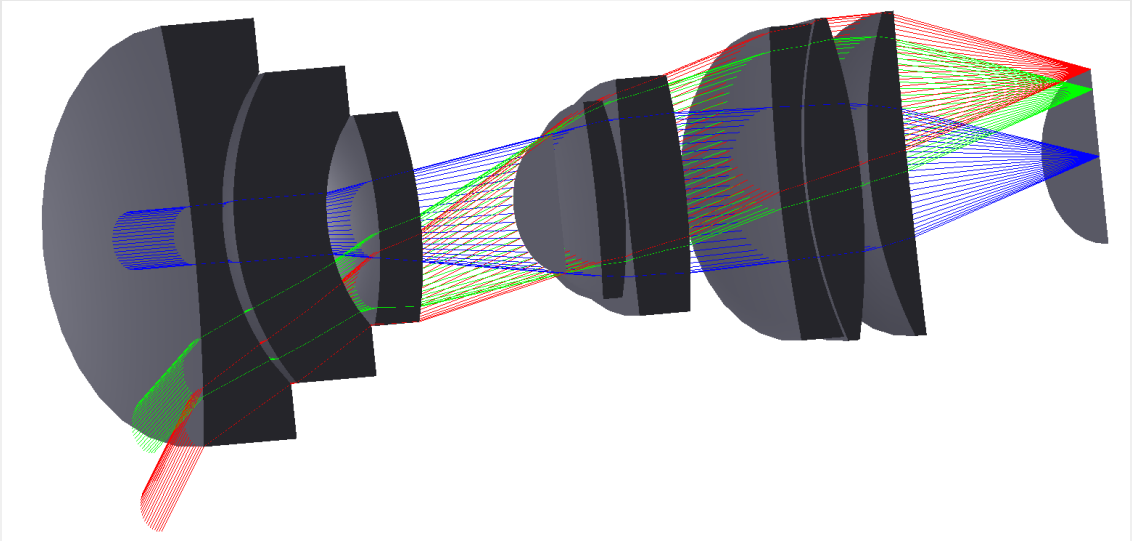
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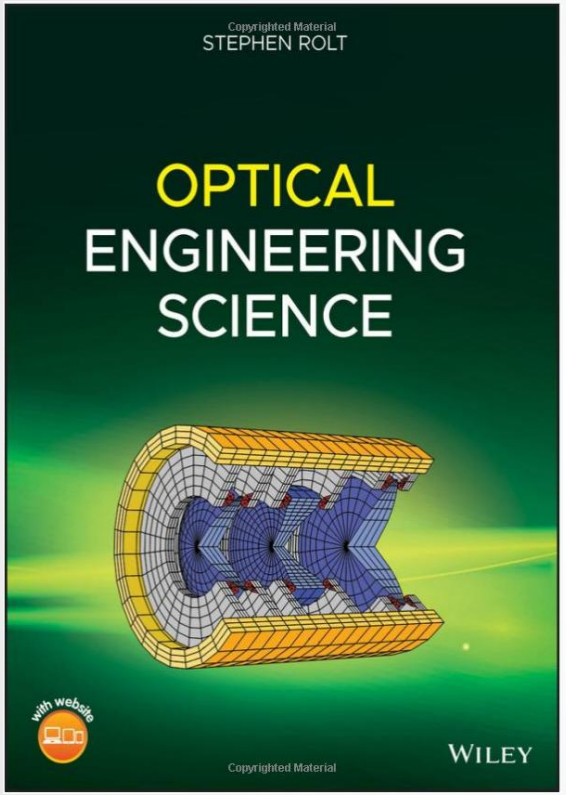
Introduction to Optical Engineering

Michaelmas Term 2026

Lecture 1 – Introductory Geometrical Optics

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Centre for Advanced Instrumentation, University of Durham



Stephen Rolt
Book released February 2020

https://cyrilbourgenot.wixsite.com/home

What is Optical Engineering?

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'Optical engineering is concerned with the practical application of optical science to the design, manufacture assembly and testing of real optical systems'

The focus of these lectures is to lay basic foundations in applied optics and to enable the more intelligent use of optical design software (Zemax).

Optical Engineering
Course Objectives

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- To understand basic geometric optics
- To recognise and be able to describe the five Gauss-Seidel aberrations
- To understand how aberrations are minimised and image quality enhanced
- To understand the impact of non-spherical surfaces in optical designs
- To be able to apply knowledge gained to further enhance intelligent use of optical design software

Optical Engineering - Outline

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- Geometrical Optics
  - General Theory
  - Fermat's Principle
  - Gaussian Optics and paraxial behaviour of Components and Surfaces
- Optical Systems and Aberrations
  - Matrix Ray Tracing
  - Stops and Pupils
  - Introduction to Monochromatic Aberrations
- Monochromatic Aberrations
  - Gauss-Seidel Aberrations
  - Behaviour of Lenses and Mirrors – Aplanatic Points
  - Impact of Pupil on Aberrations
- Aspheric Surfaces and Chromatic Aberration
  - Use of Symmetric Aspheric Surfaces
  - Dispersion and Chromatic Aberration
  - Use of Zernike Polynomials in defining OPD or WFE profiles
- Diffraction and Image Quality
  - Diffraction - 'Near Field' and 'Far Field'
  - Gaussian Beam Propagation
  - Definitions of Image Quality



Geometrical Optics
Summary

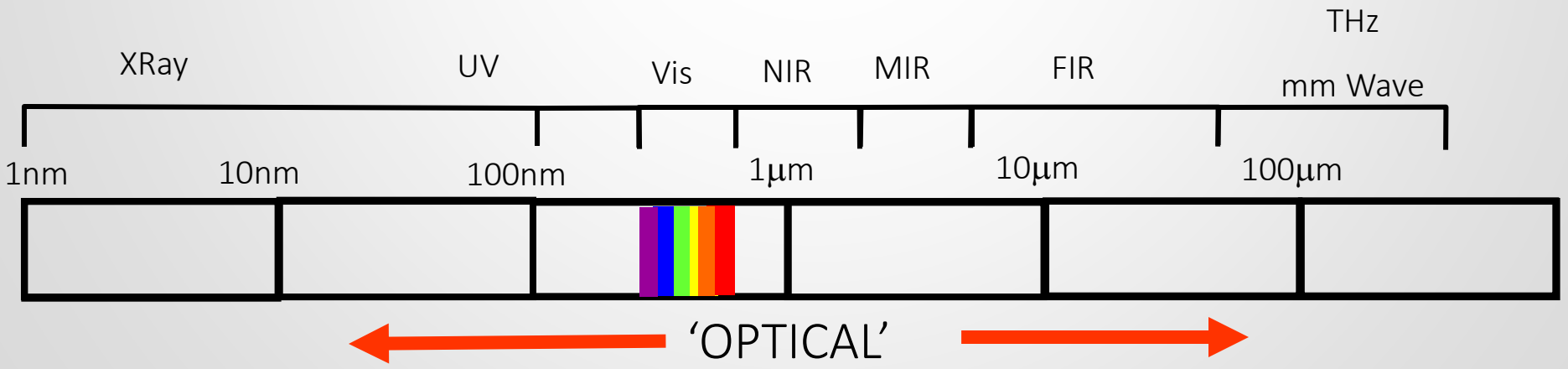
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- General description of an ideal imaging system
  - Six Cardinal Points
- Basis of geometrical optics
  - What is meant by a ray
- Fermat's Principle
  - Rays always follows the fastest route
- Reflection and refraction of rays
- Gaussian optics
  - Paraxial approximation
- Paraxial behaviour of optical surfaces

What Do We Mean by Optical?

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- Strictly speaking, ‘optical’ refers to visible light
- In this context we use the term ‘optical’ to refer to the region of the electromagnetic spectrum where the radiation can be manipulated by conventional optical “components”.
- In practice this is from the extreme ultraviolet to the infra red

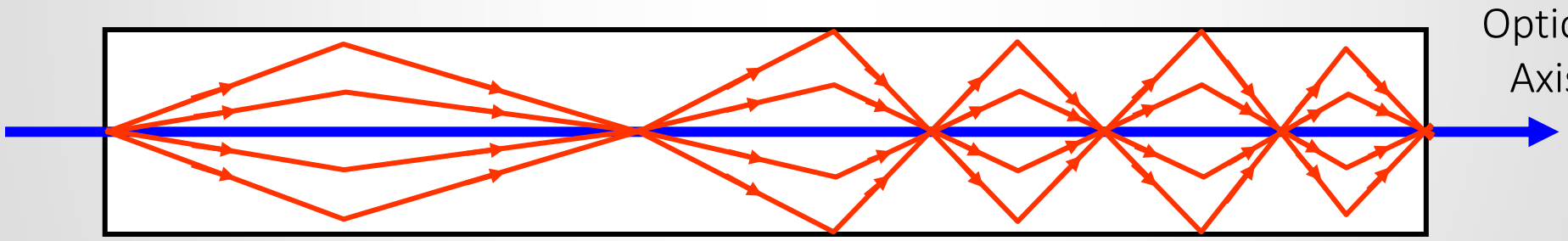


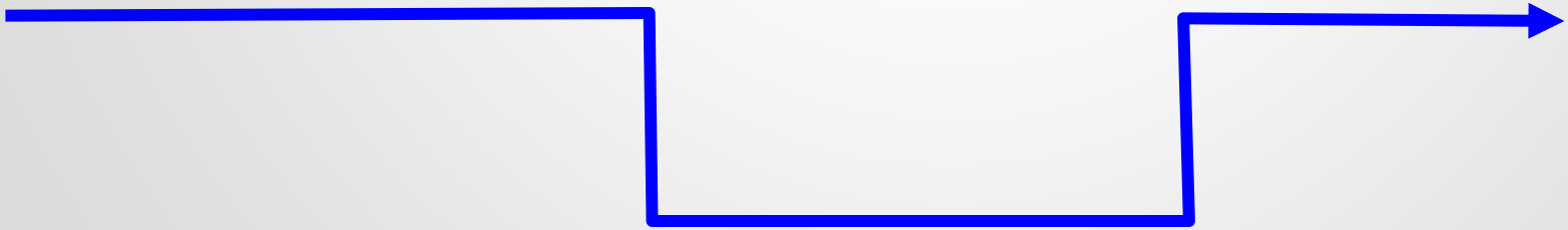
Geometrical Optics

Sequential Axial Geometry

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- ‘Light’ is directional and can be represented by ‘rays’ or arrows
  - Directional transport of ‘optical energy’ along clearly defined geometrical paths
- Light is generally confined close to a single axis or optical axis
  - Optical axis is generally axis of symmetry
    - Off axis systems

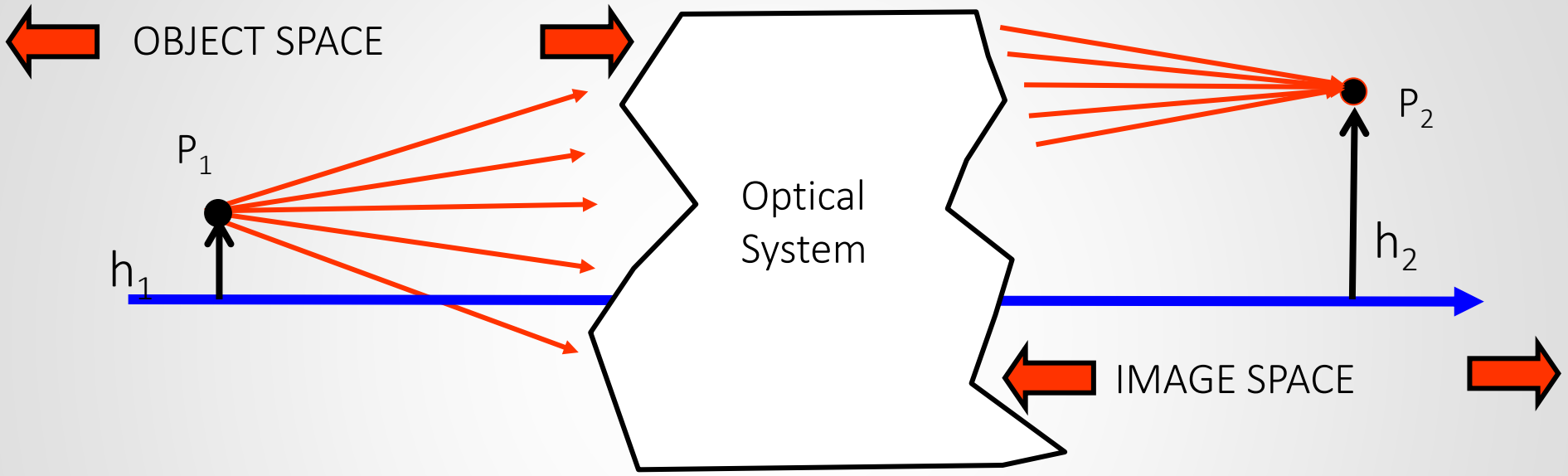




Geometrical Optics

Conjugate Points and Perfect Image Formation

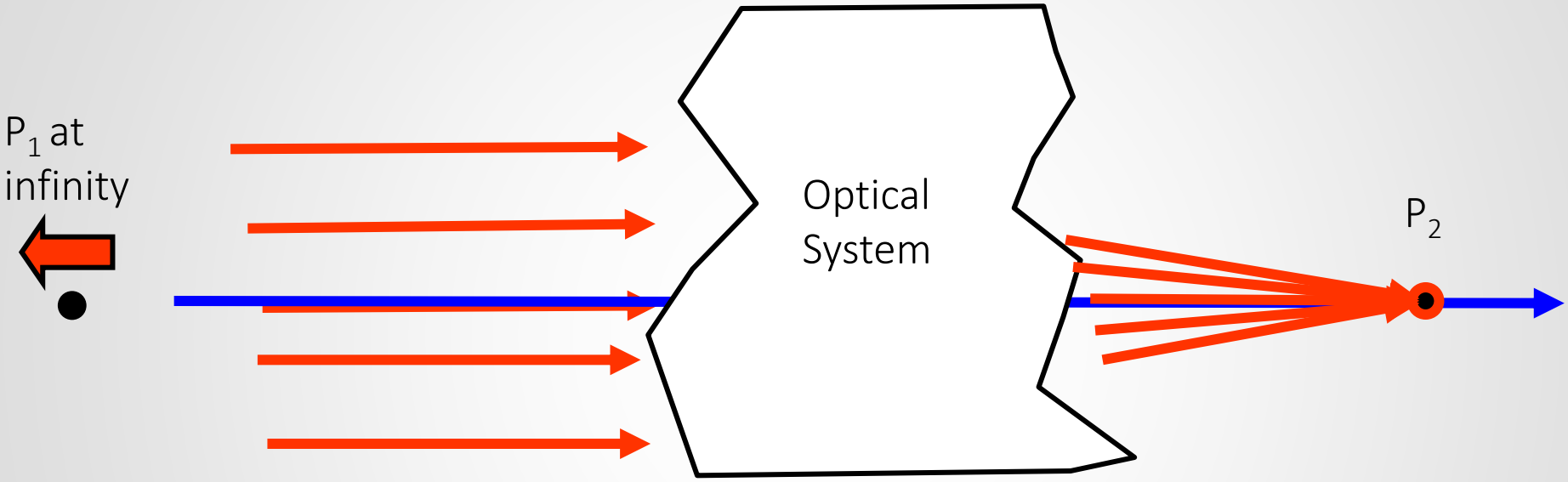
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- Two points, P_1 and P_2 have an unambiguous linear geometrical relationship – they are said to be '***conjugate***'
- System (lateral) magnification, M, is equal to h_2/h_1

Geometrical Optics

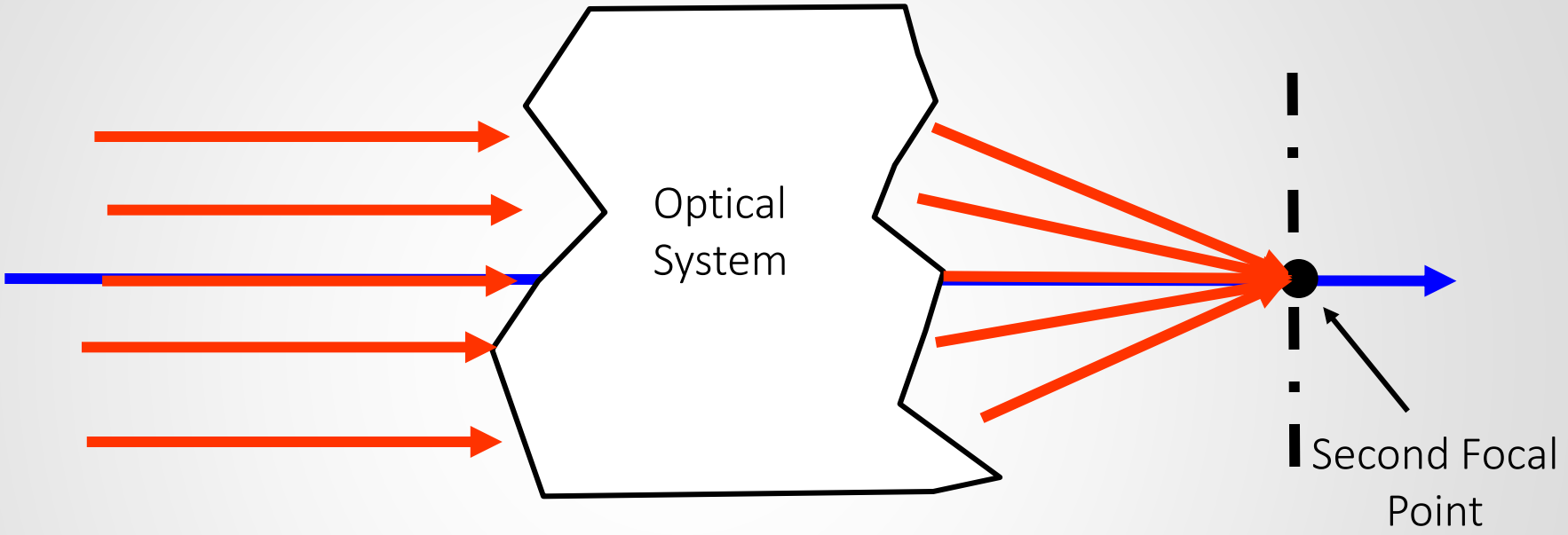
Infinite Conjugate



- Parallel rays come from a *common* point, P_1 located at infinity, 'the infinite conjugate'
- This point has a single clear conjugate point at P_2
- If rays are parallel to optical axis, P_2 lies on the axis

Perfect Optical System Definition

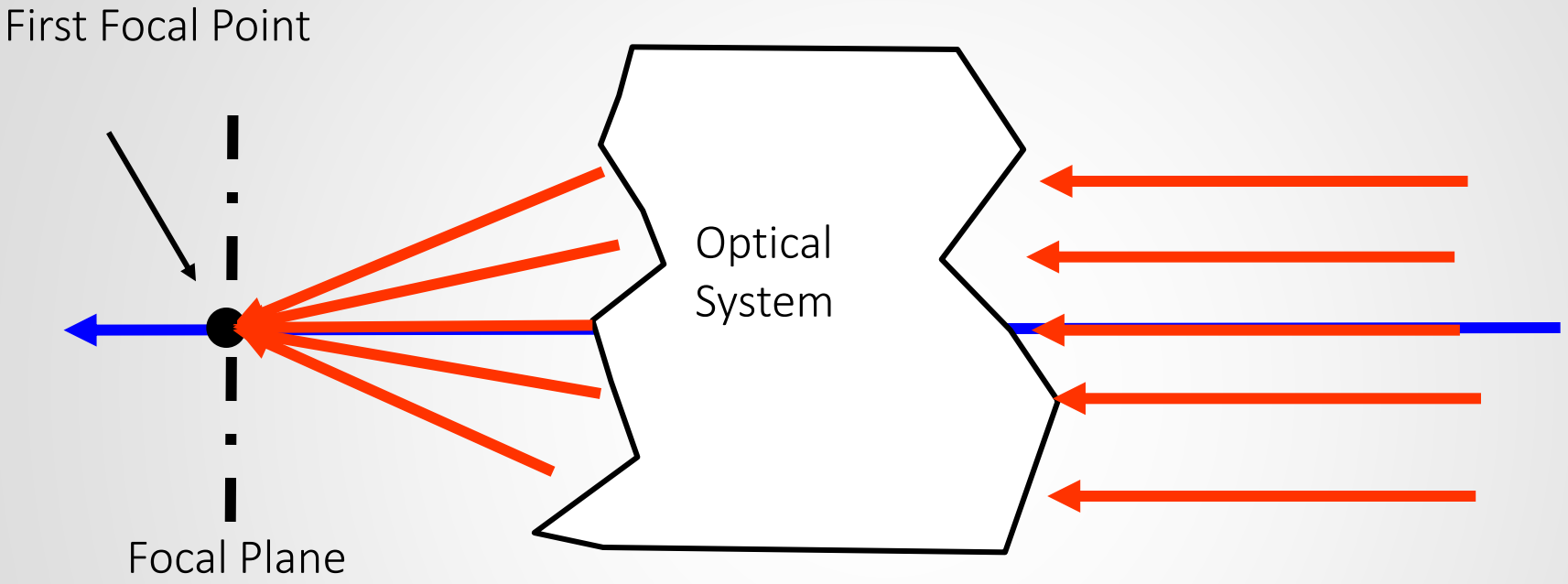
Focal Planes – 2nd Focal Point



- **Infinite conjugate** in object space
- Second focal point is conjugate point to this for on axis rays
  - Focal plane is perpendicular to optic axis and includes focal point

Perfect Optical System Definition

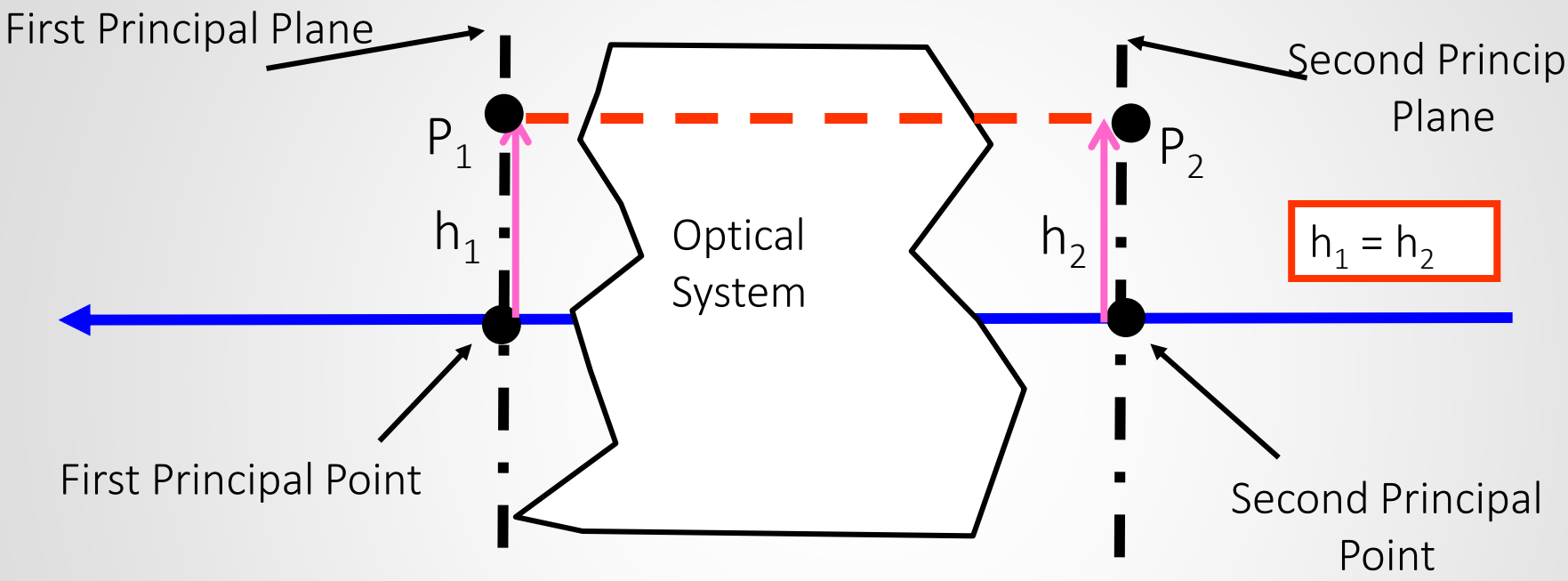
Focal Planes – 1st Focal Point



- Infinite conjugate in image space
- First focal point is conjugate point to this for on axis rays
  - Focal plane is perpendicular to optic axis and includes focal point

Perfect Optical System Definition

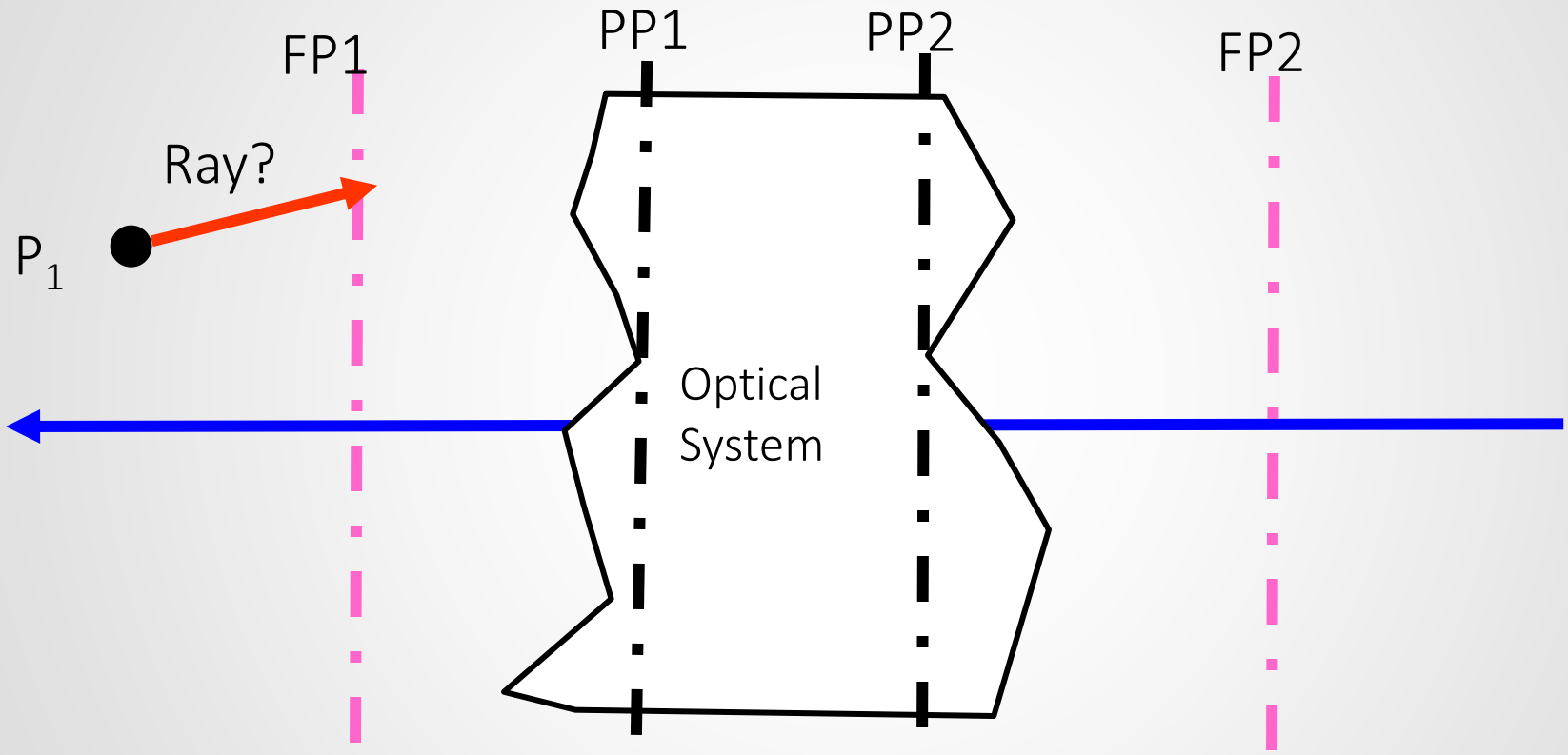
Principal Planes



- First and second principal planes are conjugate planes with positive unit magnification i.e. M = 1
- First principal plane is in object space

Perfect Optical System Definition

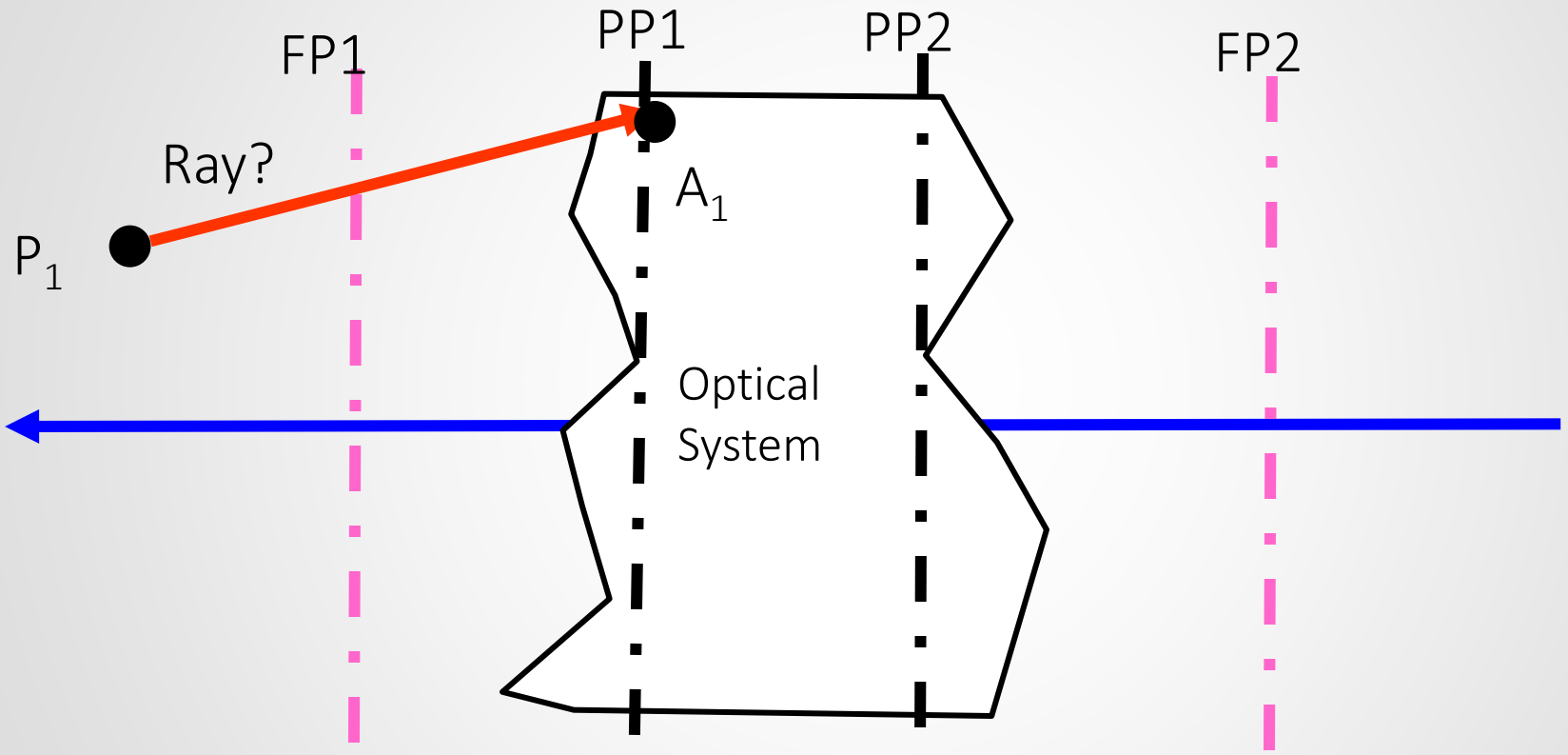
General Ray Tracing



- From given ray in object space, now trace conjugate ray in image space

Perfect Optical System Definition

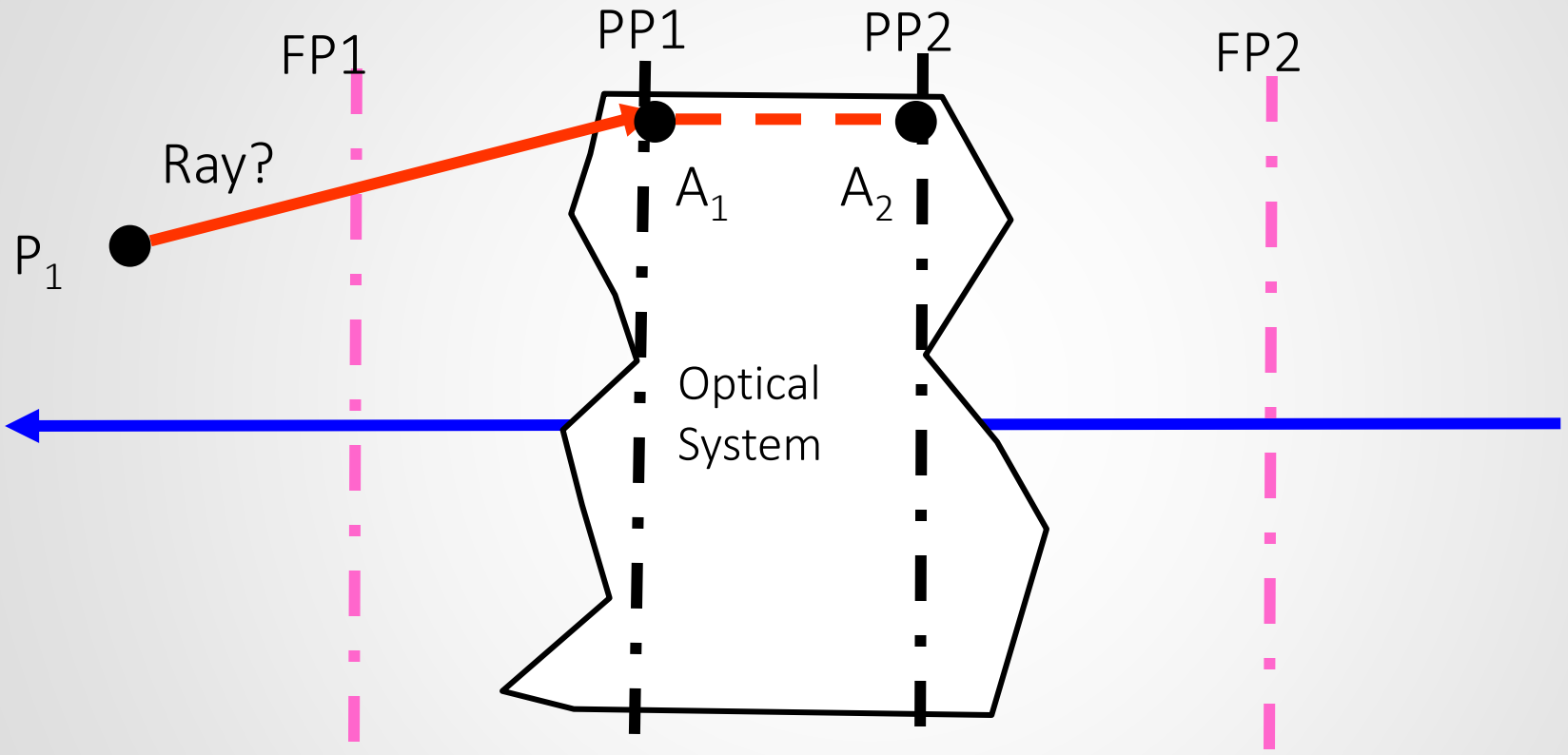
General Ray Tracing



- Project to point A_1 on first principal plane, PP1

Perfect Optical System Definition

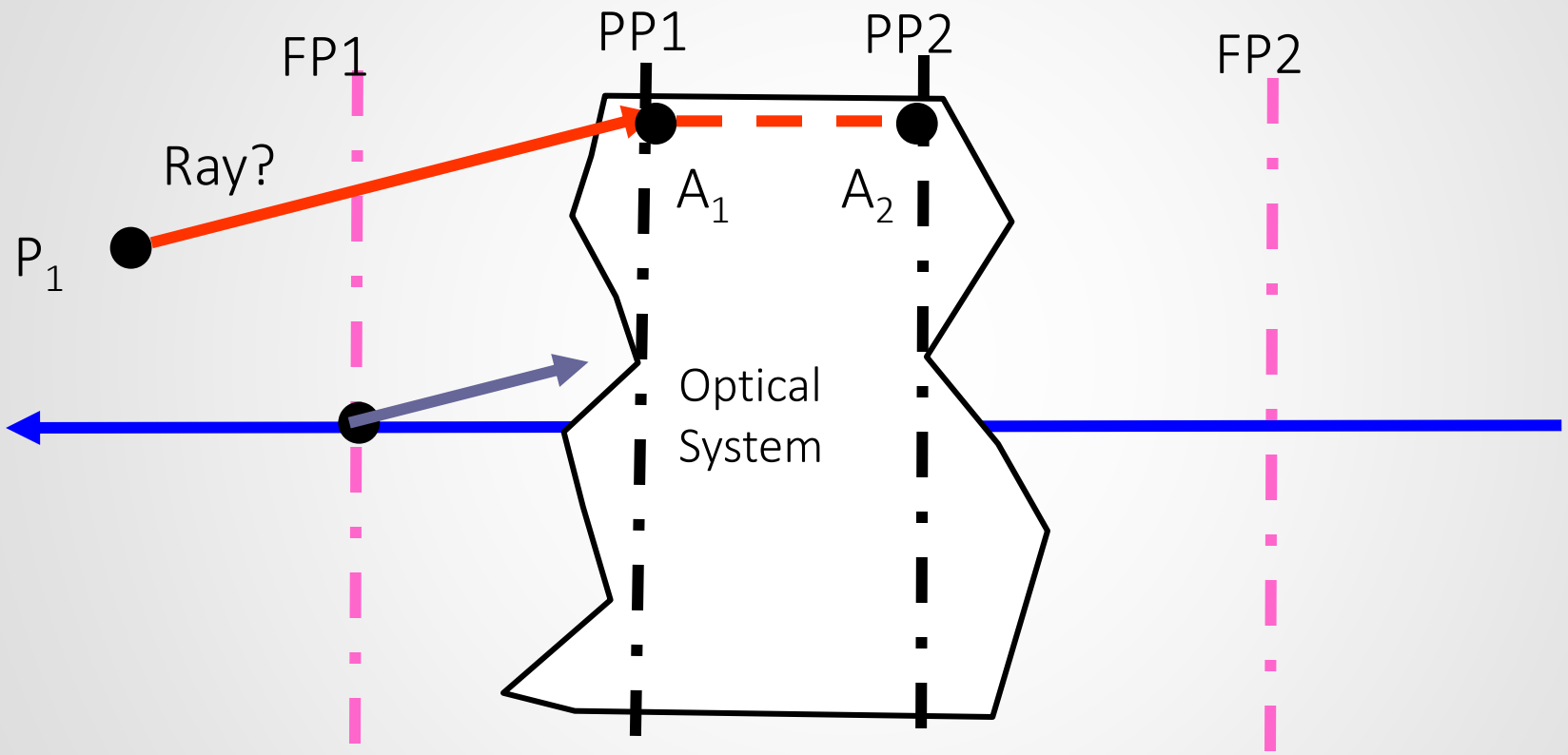
General Ray Tracing



- Conjugate to A_1 lies at A_2 on PP2 and at the same height (M=1)

Perfect Optical System Definition

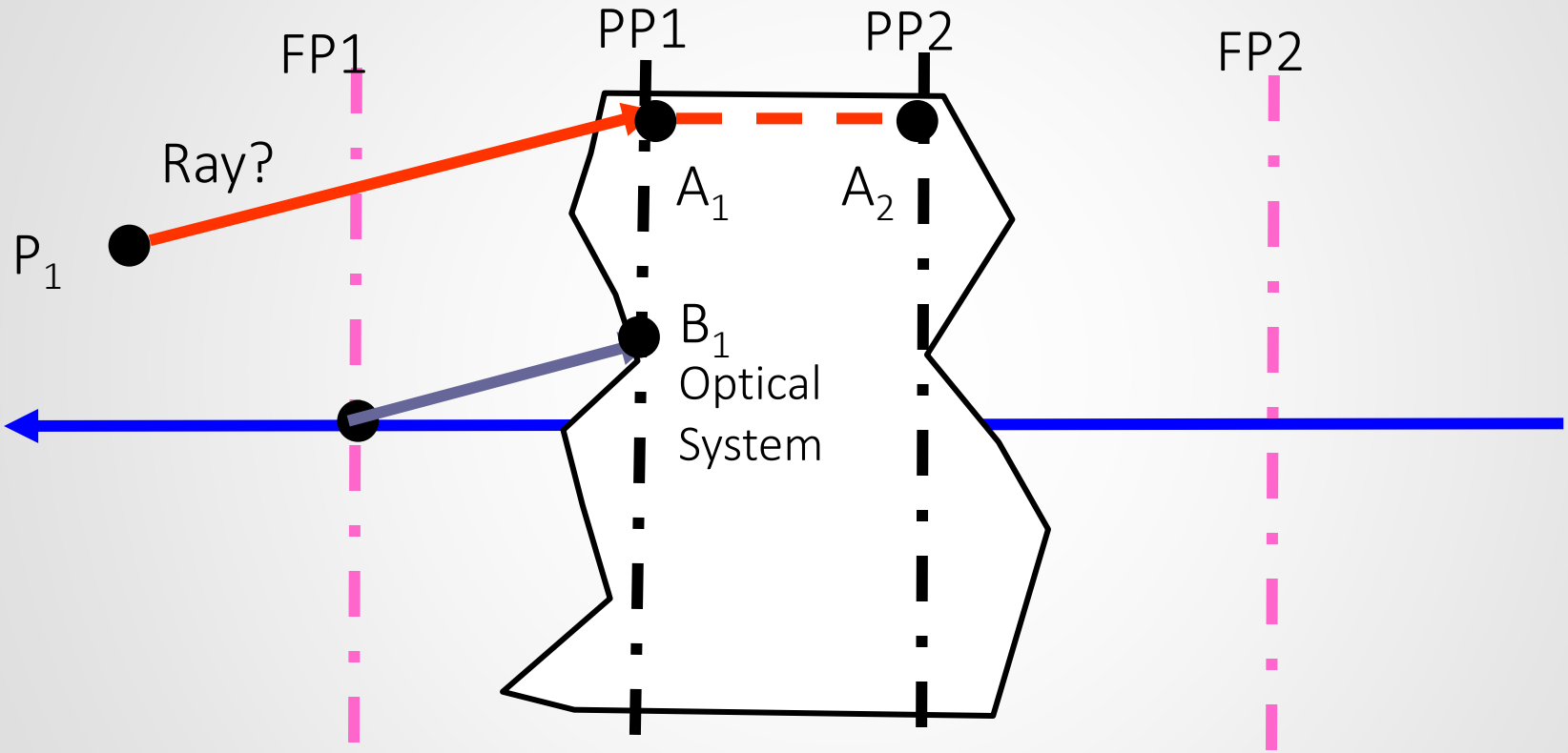
General Ray Tracing



- Trace ray parallel to object ray and originating at first focal point

Perfect Optical System Definition

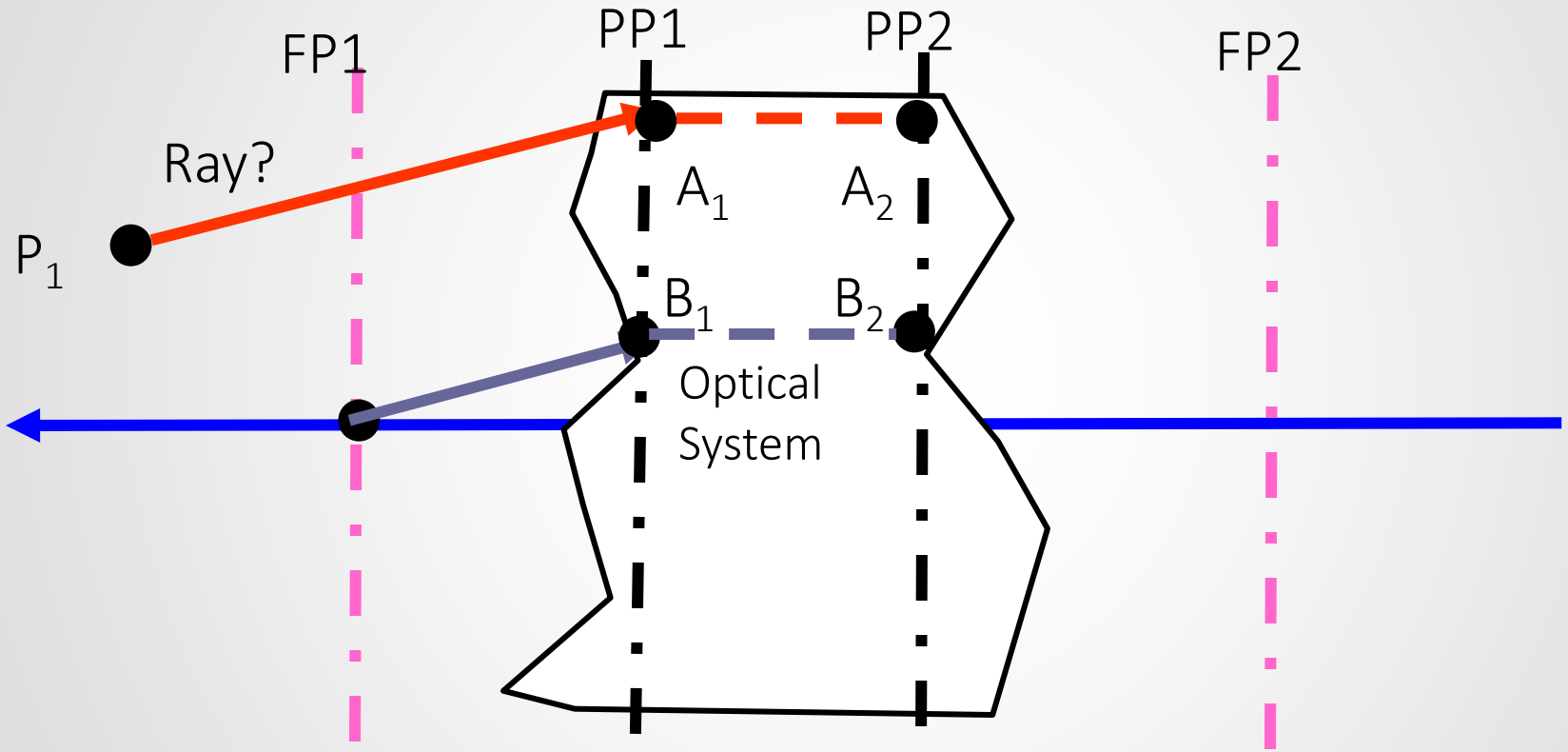
General Ray Tracing



- Trace dummy ray to first principal plane at B_1

Perfect Optical System Definition

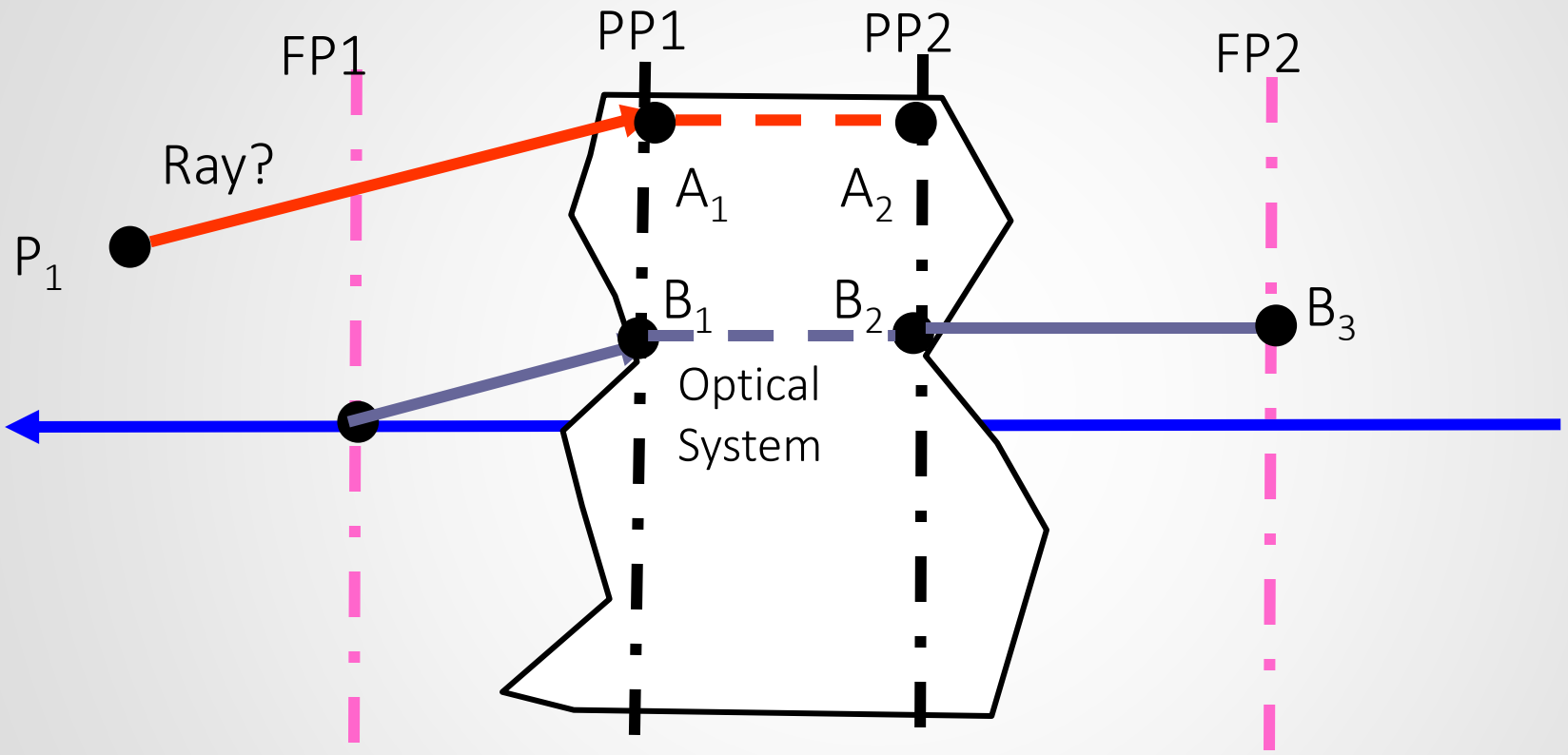
General Ray Tracing



- Conjugate point to B_1 is B_2

Perfect Optical System Definition

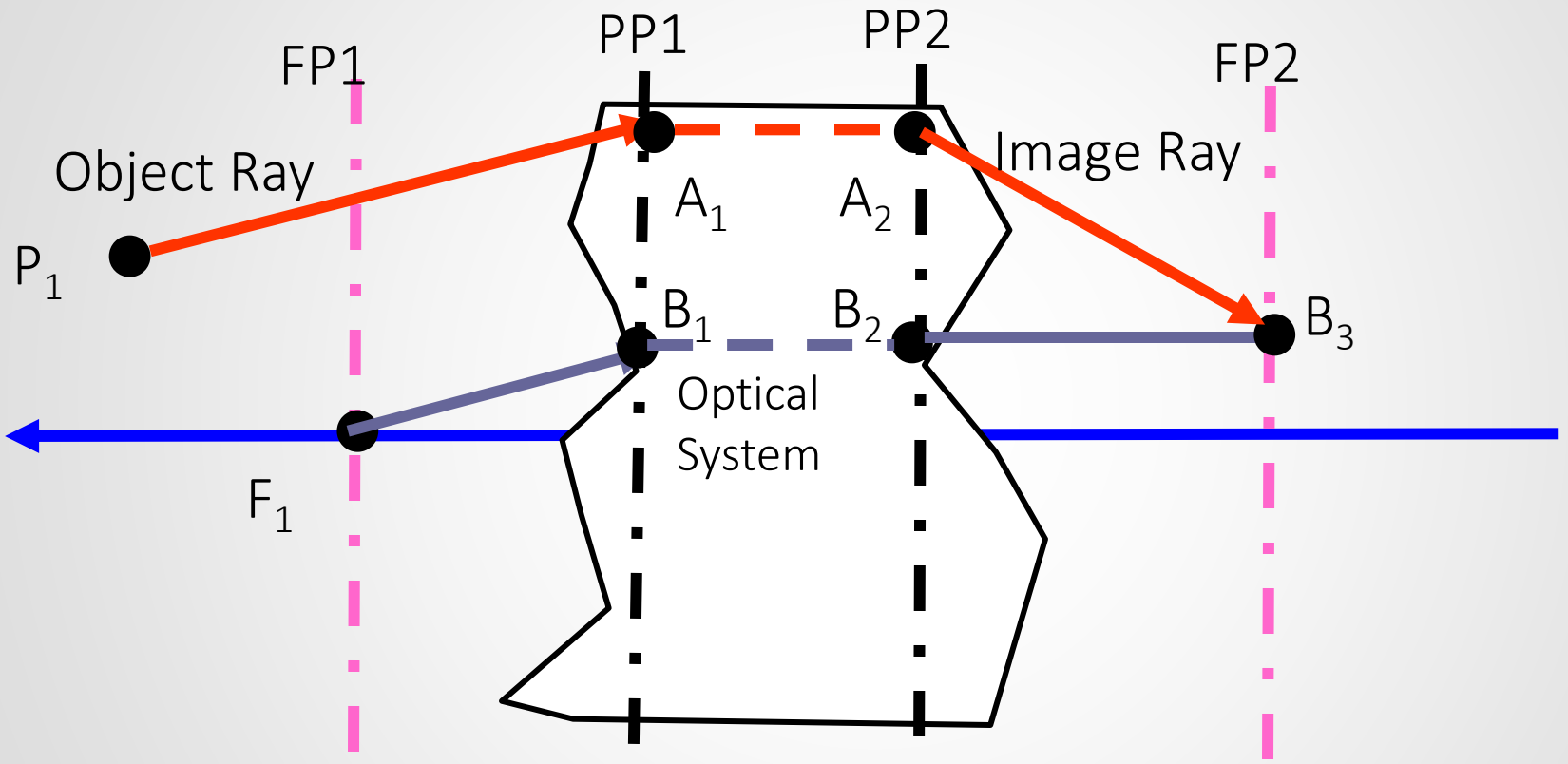
General Ray Tracing



- By definition the conjugate to rays coming from the first focal point are rays parallel to the optic axis (infinite conjugate)

Perfect Optical System Definition

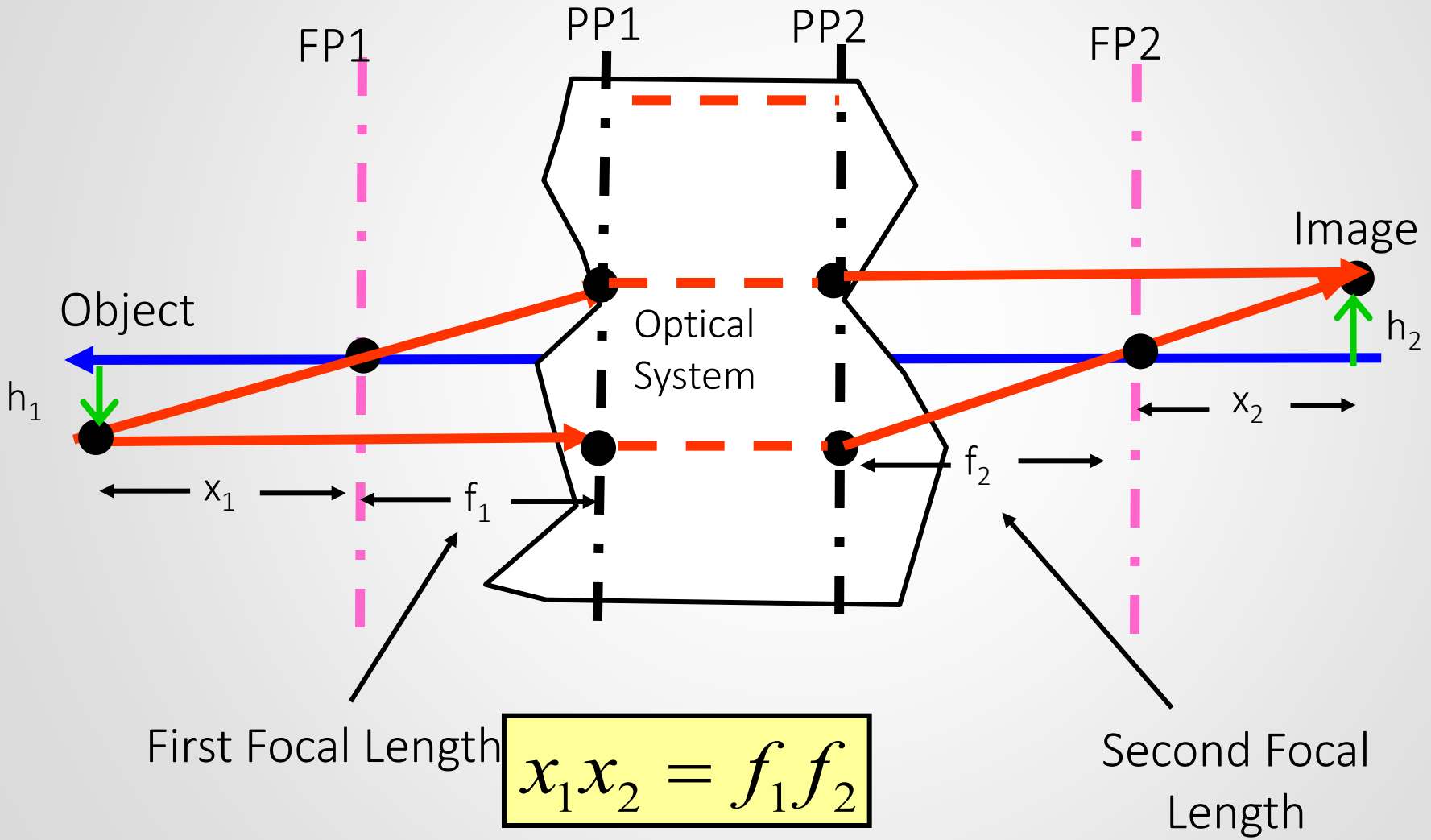
General Ray Tracing



- The parallel rays P_1:A_1 and F_1:B_1 emerge from an off axis point at infinity. The conjugate poin to this must lie on FP2, i.e at B_3

Perfect Optical System Definition

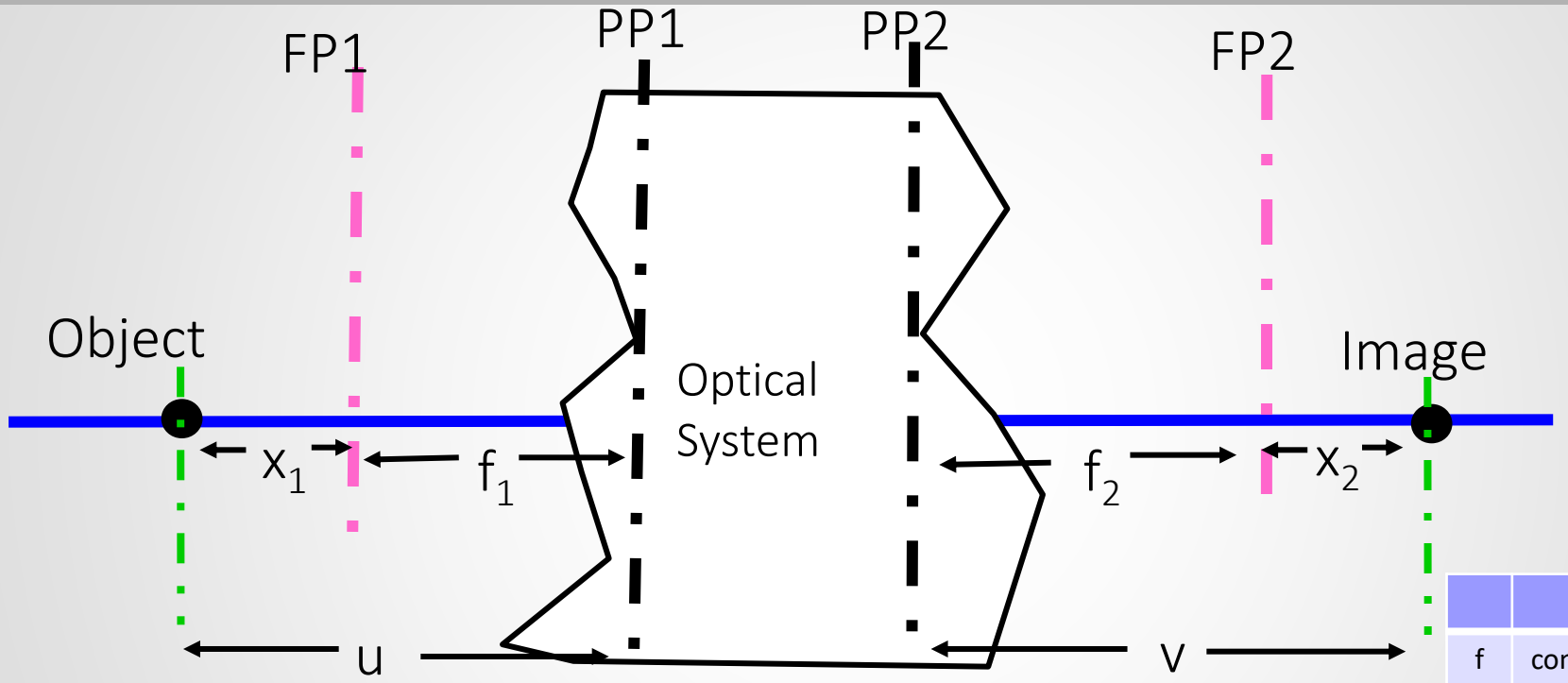
Newton's Equation



$$x_1 x_2 = f_1 f_2$$

Perfect Optical System Definition

Object and Image Distances



	+	-
f	converging	diverging
u	In front of lens	Behind lens
v	Behind lens	In front of lens

- Object distance, u, is distance from object to first principal plane
- Image distance, v, is distance from object to second principal plane

$$\frac{1}{u}+\left(\frac{f_2}{f_1}\right)\frac{1}{v}=\frac{1}{f_1}$$

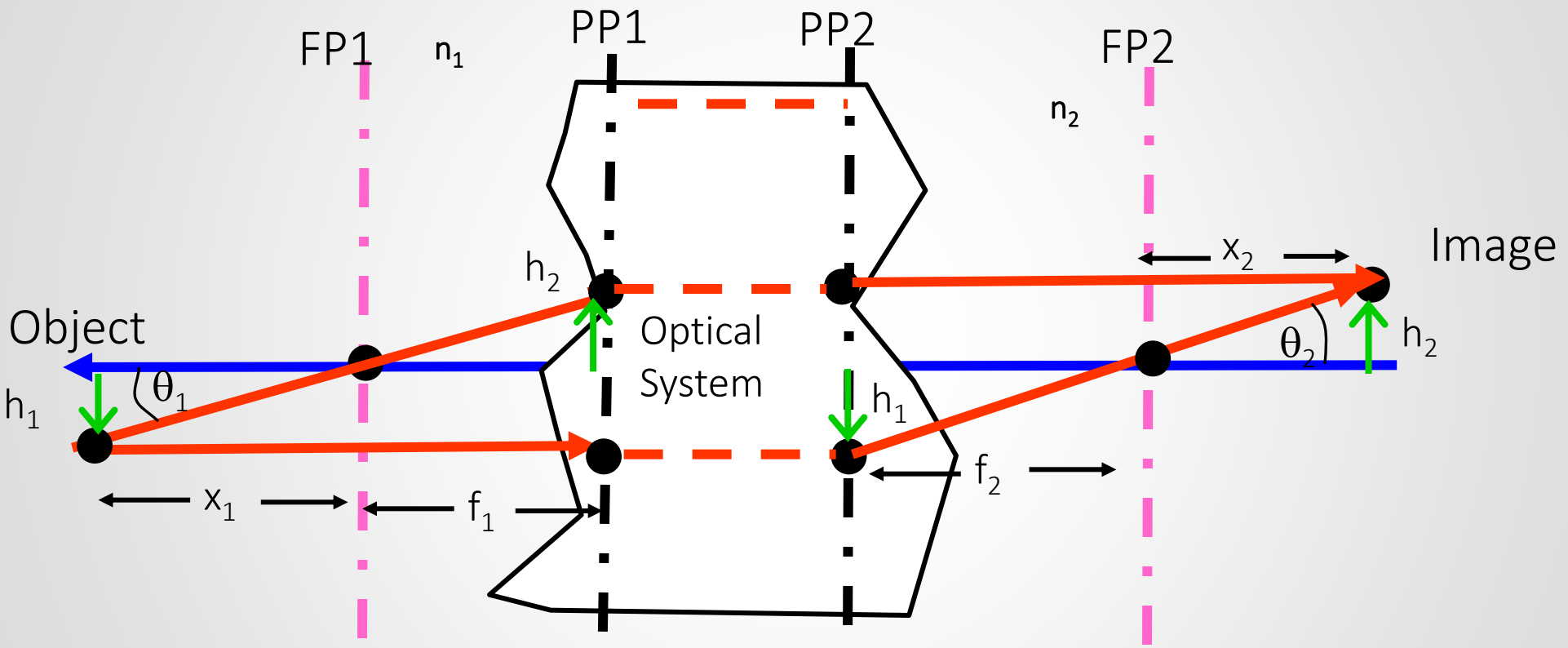
- From Newton's Equation

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

- If $f_1 = f_2 = f$

Perfect Optical System Definition

Helmholtz Equation



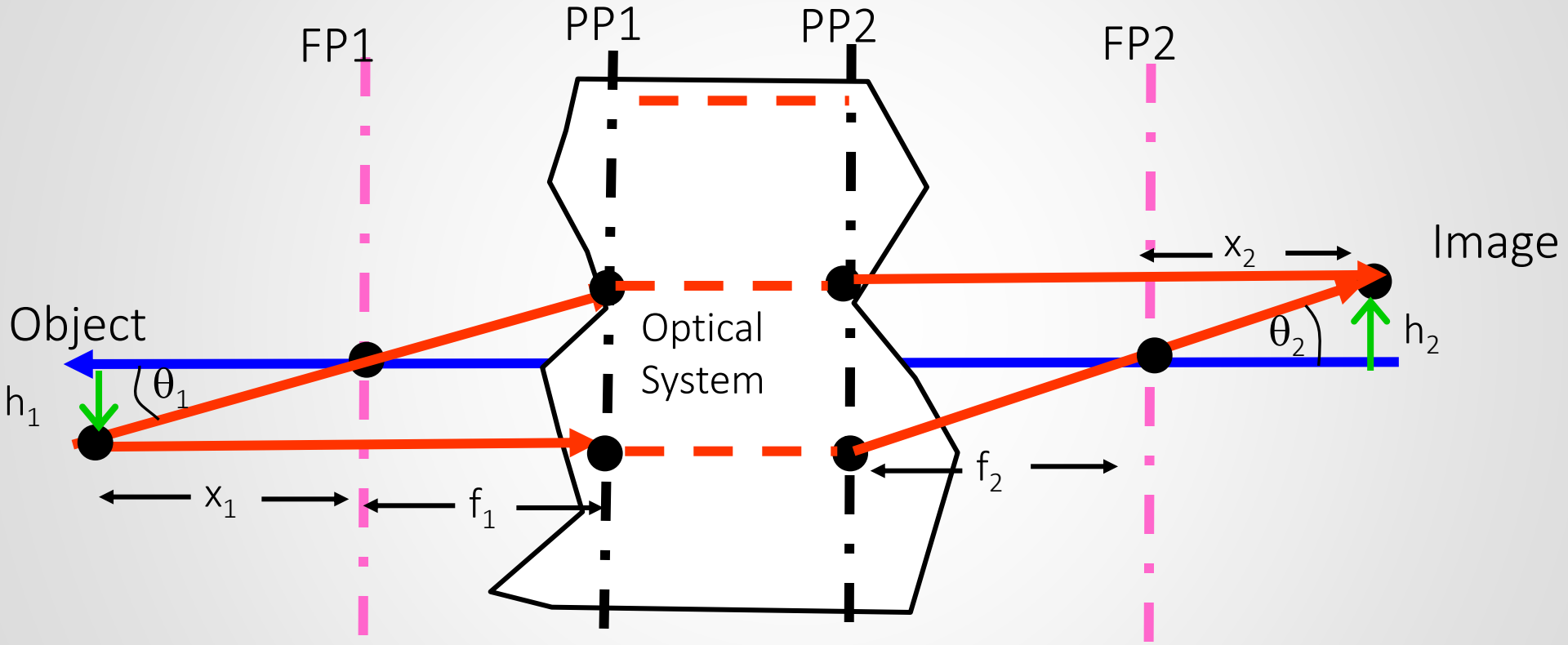
$$\frac{n_1}{n_2} = \frac{f_1}{f_2}$$

$$h_1 n_1 \tan\theta_1 = h_2 n_2 \tan\theta_2$$

Helmholtz equation

Perfect Optical System Definition

Transverse Magnification



$$M = \frac{h_2}{h_1} = -\frac{f_1}{x_1} = -\frac{x_2}{f_2}$$

Perfect Optical System Definition

Angular Magnification

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- Helmholtz equation dictates that angular magnification is inversely proportional to lateral magnification

$$h_1 n_1 \tan\theta_1 = h_2 n_2 \tan\theta_2$$

- Angular magnification

$$A = \frac{\tan\theta_2}{\tan\theta_1}$$

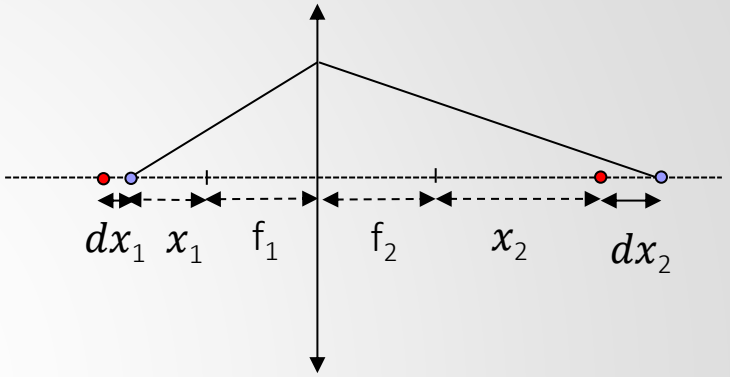
- From Helmholtz

$$A = \frac{n_1}{n_2}\frac{1}{M}$$

Perfect Optical System Definition

Longitudinal Magnification

- Longitudinal Magnification $L = \frac{dx_2}{dx_1}$

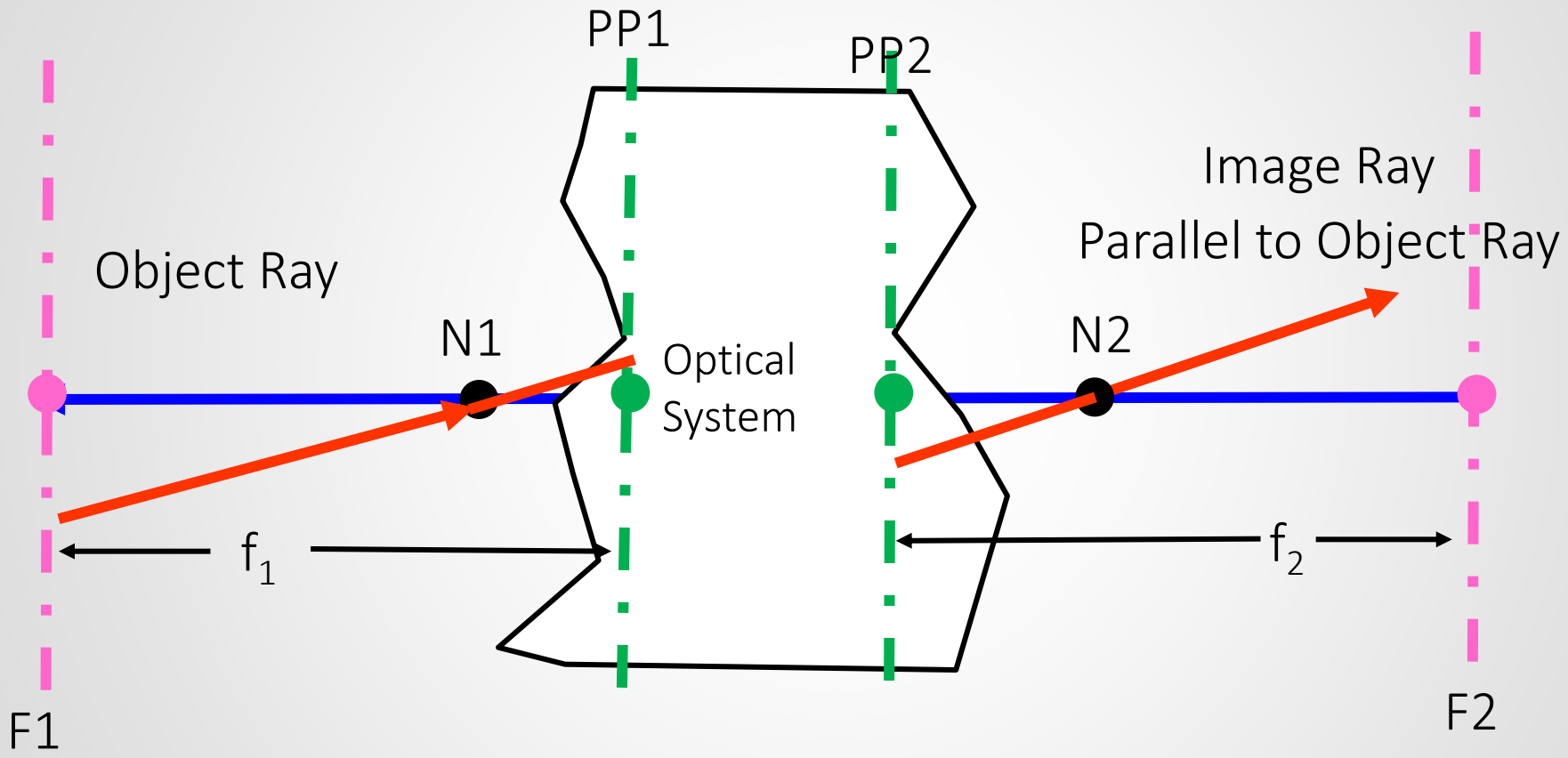


- From Newton's Equation $x_2 = \frac{f_1 f_2}{x_1}$

$$\frac{dx_2}{dx_1} = -\frac{f_1 f_2}{x_1^2} = -\frac{f_2}{f_1}\left(\frac{f_1}{x_1}\right)^2 = -\frac{n_2}{n_1}M^2$$

Perfect Optical System Definition

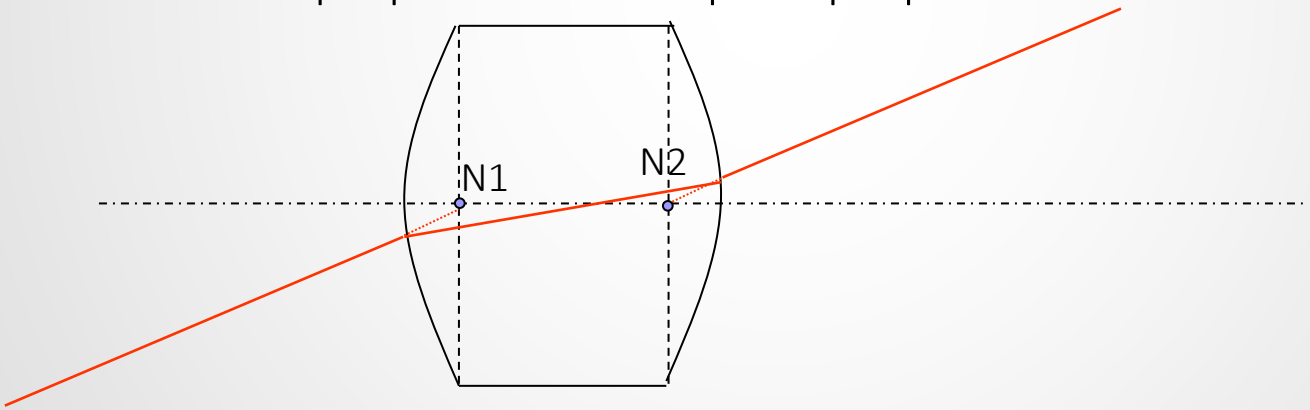
Nodal Points



- The nodal points are two conjugate points, N1 and N2, where the rays in the object and image spaces are parallel

Definition of Optical System
Cardinal Points

- **Cardinal Points**
  - Two Focal Points
  - Two Principal Points
  - Two Nodal Points
    - For most purposes same as principal points



- **Provides complete definition of 'perfect' optical system**

Origin of 'Rays'

Wave Equation

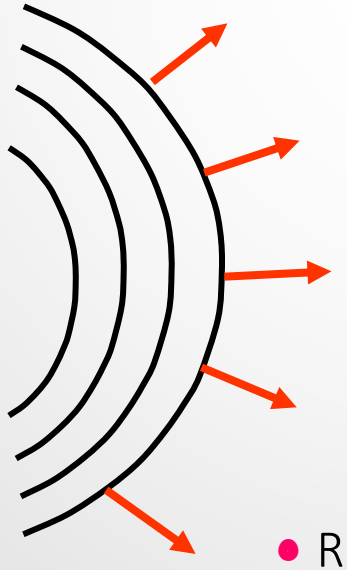
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- Light is a wave disturbance
  - Where do rays come from?
- Maxwell's Equation (*homogeneous wave equation)*

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}=\mu\mu_0\varepsilon\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$Ae^{i[k_0 S(x,y,z)-\omega t]}$$

Wave Disturbance



$$k_0 = 2\pi/\lambda$$

$$\omega = 2\pi f$$

$S(x,y,z)$: Spatial phase function (eikonal)

$S(x,y,z) = z$ (plane wave)

$S(x,y,z) = \sqrt{x^2+y^2+z^2}$ spherical wave

- Rays are the normals to the surfaces of equal phase

Origin of 'Rays'

The *Ray Equation*

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- Eikonal Equation provides a bridge between electromagnetic wave theory and ray optics.
- S(r) represent surfaces of equal phase, S is called the Eikonal (from Greek (έίκων = eikon → image)).

$$\left(\frac{\partial S(r)}{\partial x}\right)^2 + \left(\frac{\partial S(r)}{\partial y}\right)^2 + \left(\frac{\partial S(r)}{\partial z}\right)^2 = n^2(x, y, z)$$

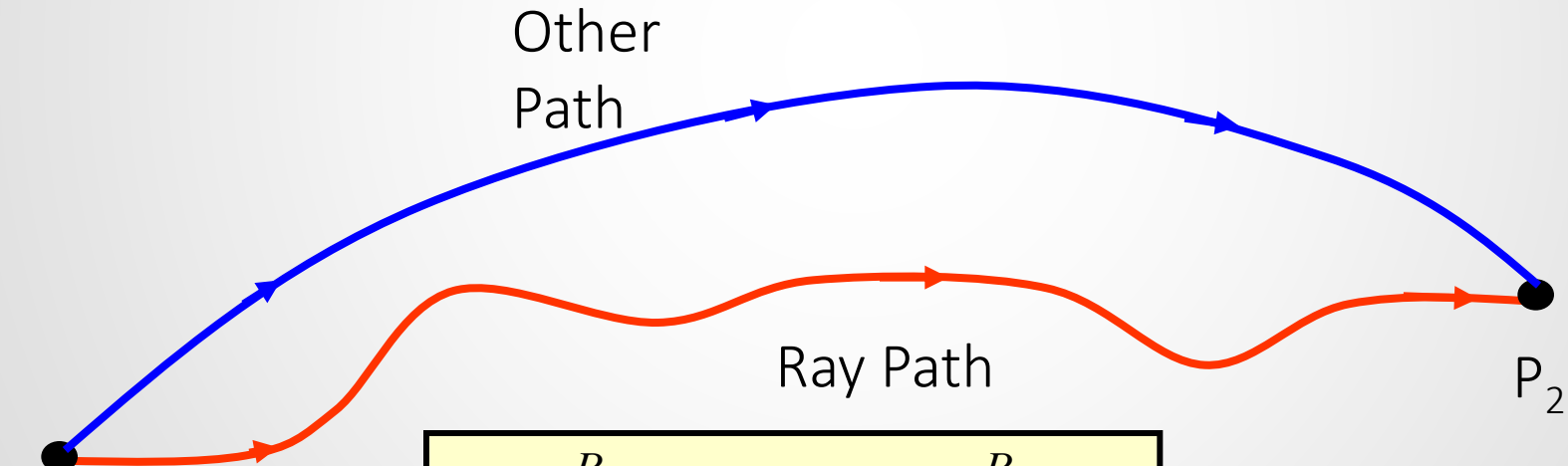
- Derivation assumes S(r) varies slowly compared to wavelength, wavelength is much smaller than distance of propagation
- Breakdown in this approximation represents the limit of geometrical optics
  - Physical optics approach (i.e. diffraction) required

Ref : solo Hermelin (Foundation of Geometrical Optics)

Fermat's Principle

Shortest Optical Path

- The path of a ray between two points always minimises the optical path or the time taken
- Direct consequence of Maxwell's Law via. Eikonal Equation
- n (*refractive index*) = speed of light in vacuum / speed in medium = c / v

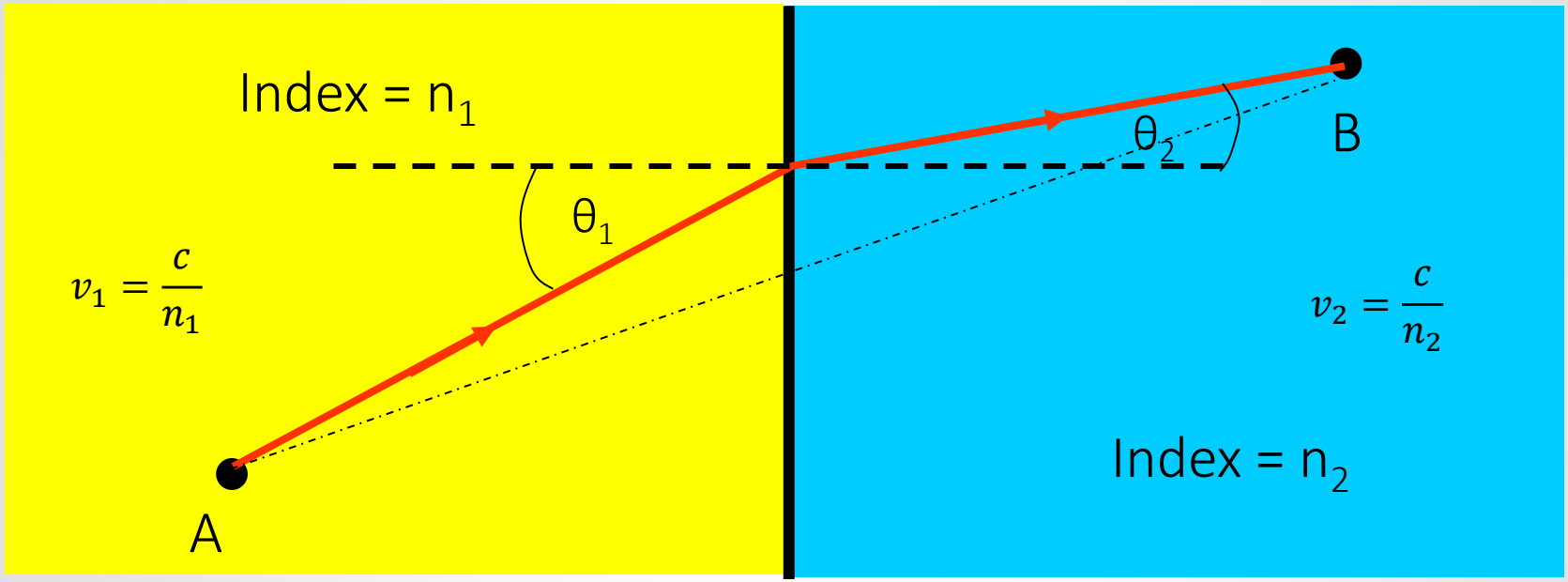


$$Ray\int_{P_1}^{P_2} nds < Other\int_{P_1}^{P_2} nds$$

- For all other paths

Fermat's Principle

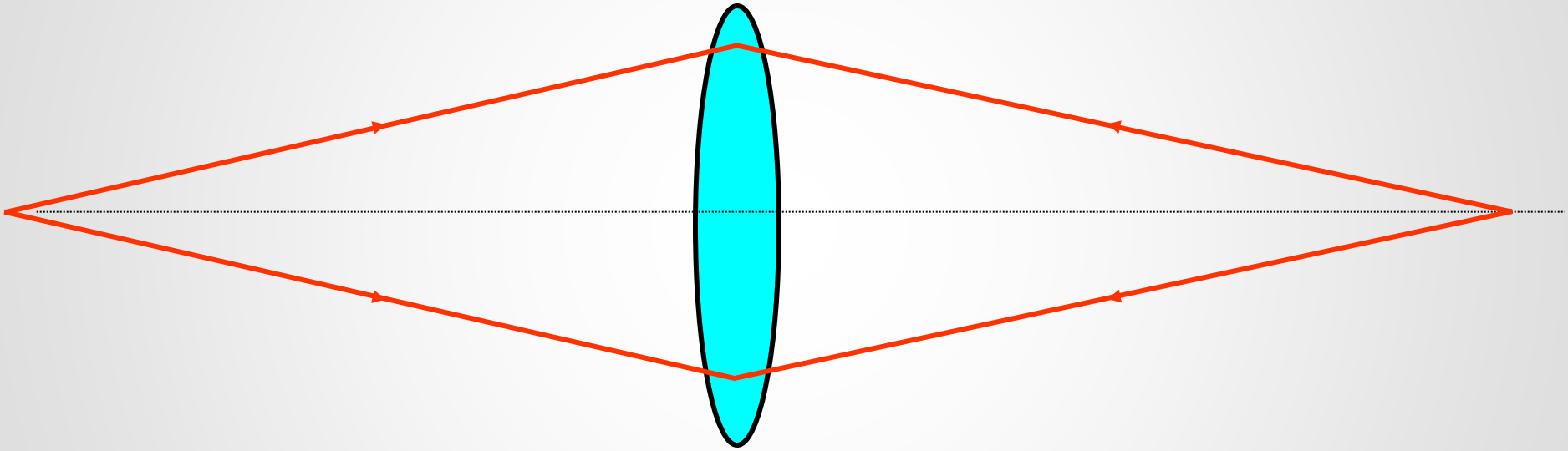
Snell's Law for Refraction



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Geometrical Optics

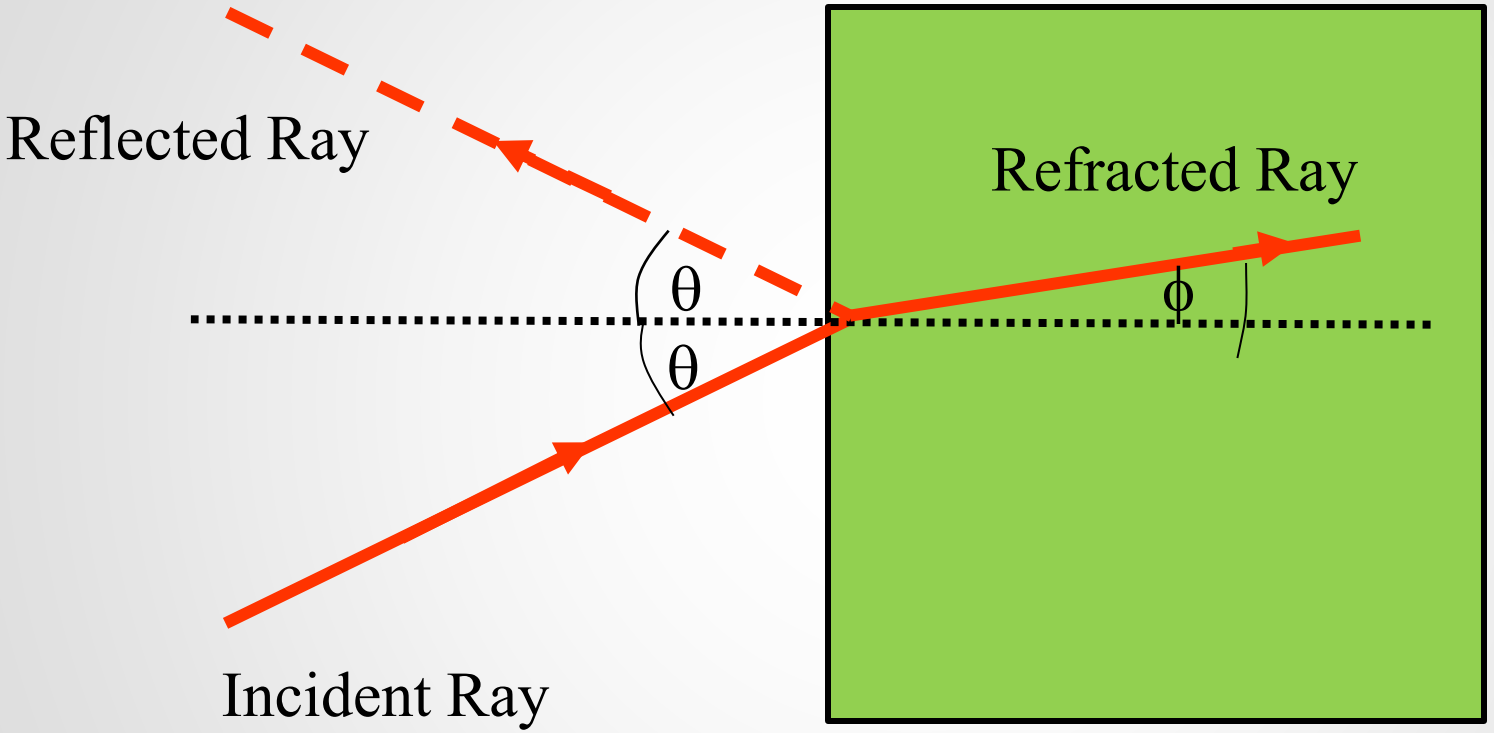
Underlying Processes & Components



- Fermat's principle dictates that rays travel in a straight line in a homogenous material
- At the boundary of different materials, ray propagation is dictated by Snell's Law

Underlying Processes

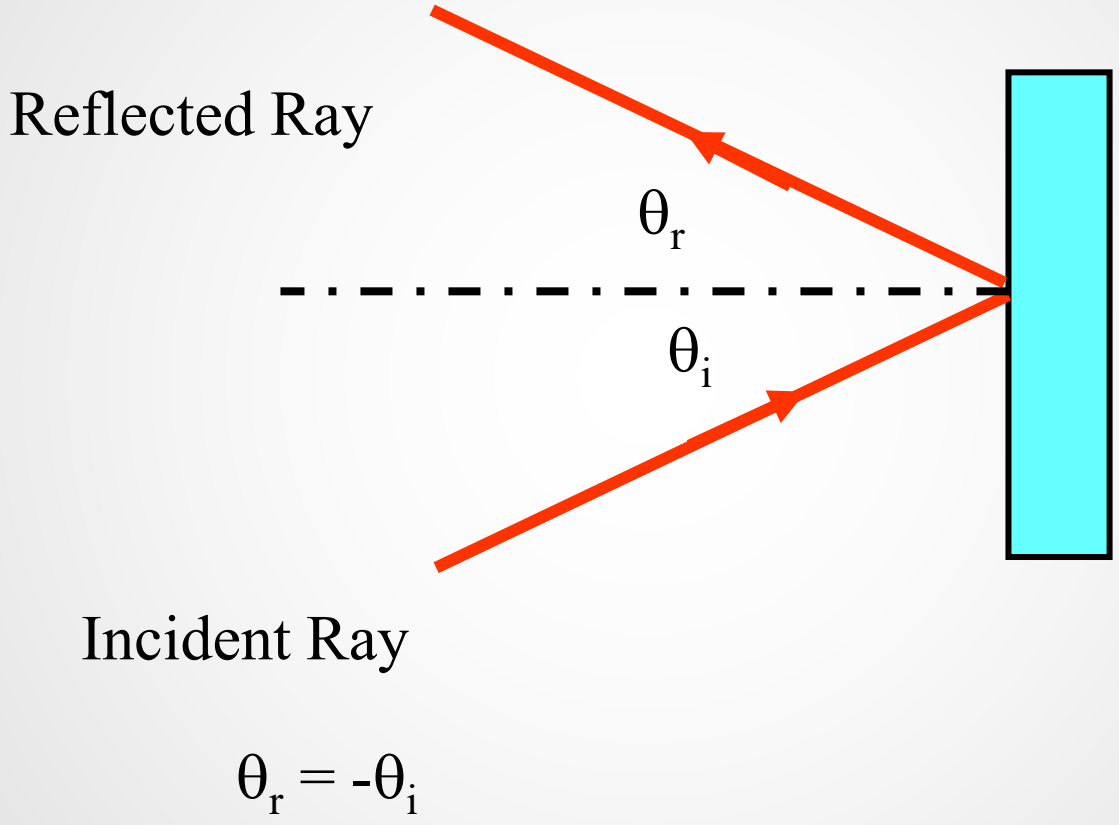
Refraction



- Refractive index = n = (Speed of Light in Vacuum)/(Speed of Light in the Medium)
- Snell's Law: $\sin\theta = n\sin\phi$

Underlying Processes

Reflection

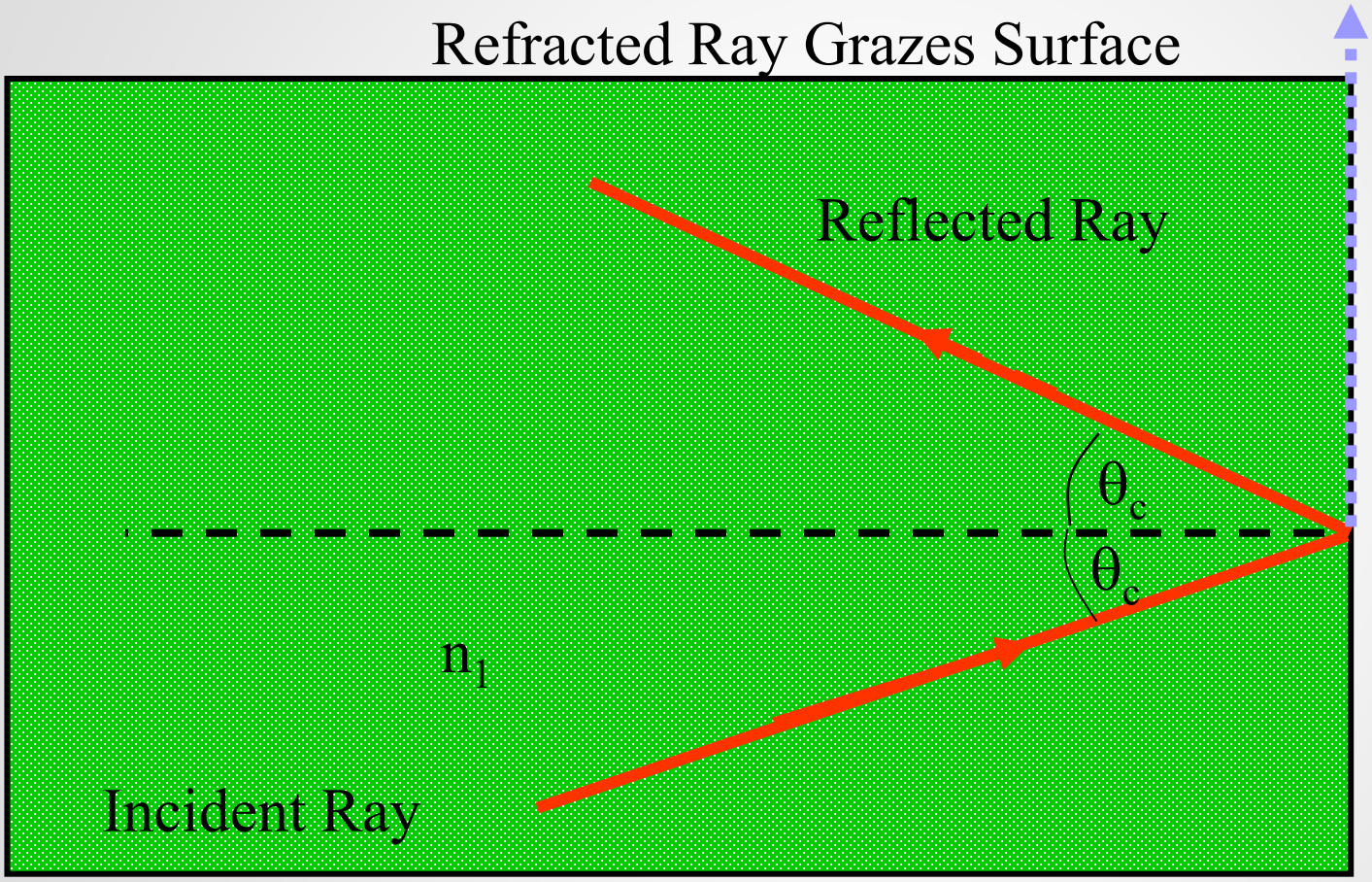


$$\theta_r = -\theta_i$$

- Could be seen as refraction with n = -1

Underlying Processes

Total Internal Reflection



- $\theta > \theta_c$ - no refracted ray (100% reflected)

$$\sin\theta_c = \frac{n_2}{n_1}$$

Paraxial Approximation

Gaussian Imaging

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- Analysis of refraction much easier if we assume

$$\sin\theta << 1 \quad \text{and} \quad \theta \approx \sin\theta \approx \tan\theta$$

- Snell's Law becomes

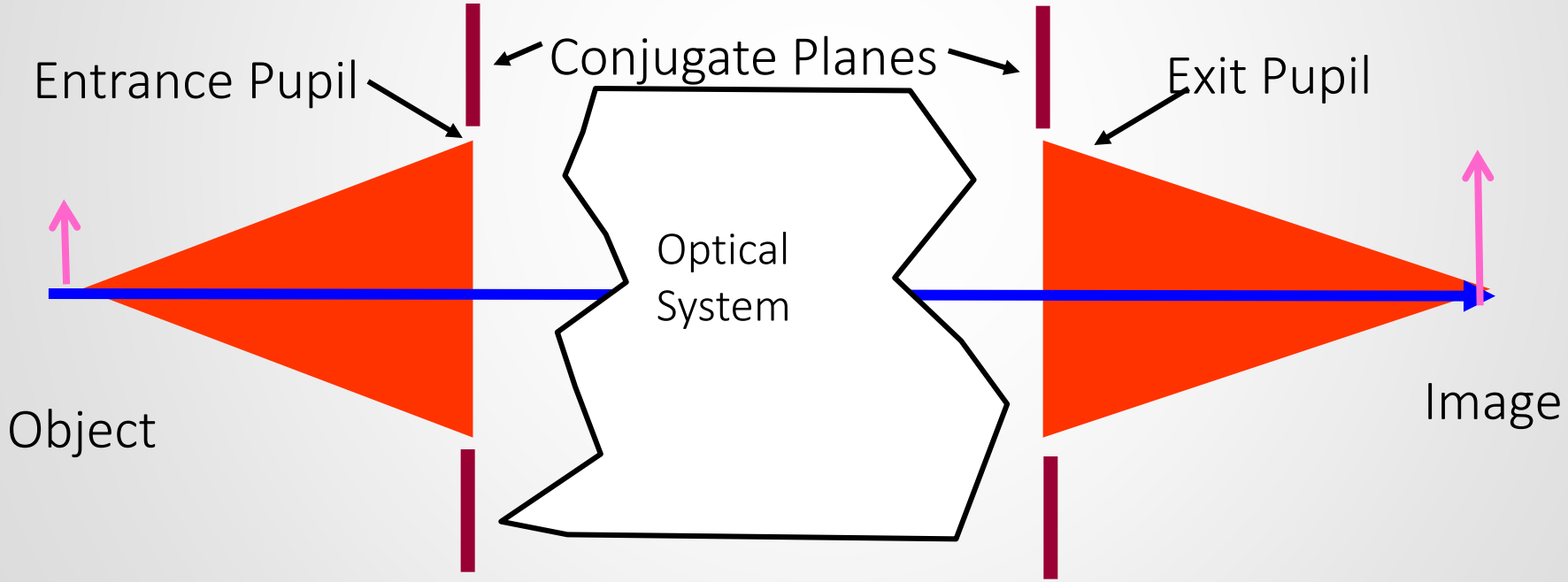
$$\theta = n\phi$$

- Now easier to formulate Snell's Law for curved surfaces
- Valid for small θ – rays very close to the optical axis
- This approximation leads to perfect image formation
  - Can apply previous 'ideal analysis' to real systems
- Approximation breaks down for large angles and leads to imperfections or aberrations

Paraxial Approximation

Restriction of Rays by Aperturing

- To restrict rays passing through an optical system to those close to the axis, rays must be rejected by aperturing
  - True of all systems not just 'paraxial' or ideal systems



- The entrance pupil is a physical aperture imaged in object space

Paraxial Approximation

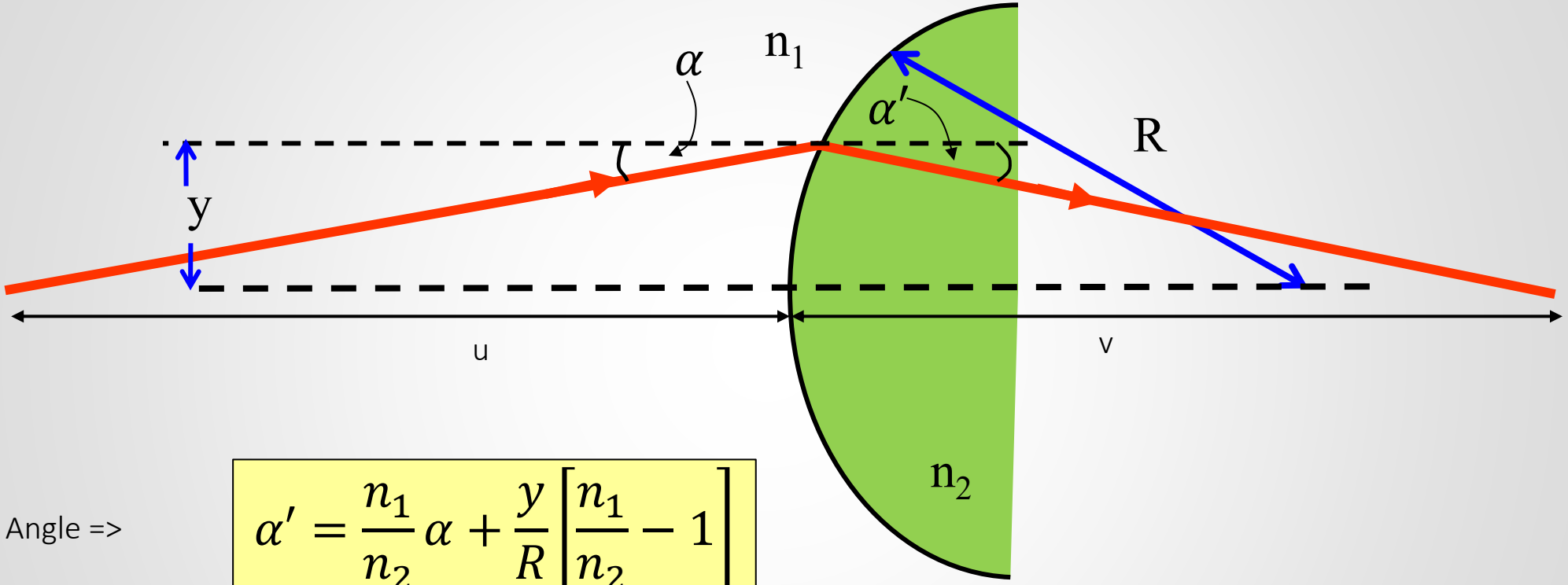
Real System Components

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- Using previous analysis we can ‘populate’ ideal system
- Refractive and reflective surfaces
  - Planar and curved surfaces
- Simple lens components
- Use paraxial approximation
- Derive behaviour of system elements using Snell’s Law etc.

Refraction at Curved Surfaces

Paraxial Treatment



Angle =>

$$\alpha' = \frac{n_1}{n_2}\alpha + \frac{y}{R}\left[\frac{n_1}{n_2} - 1\right]$$

Distance =>

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{(n_2 - n_1)}{R}$$

Use the gradient β :

$y = R.\beta$

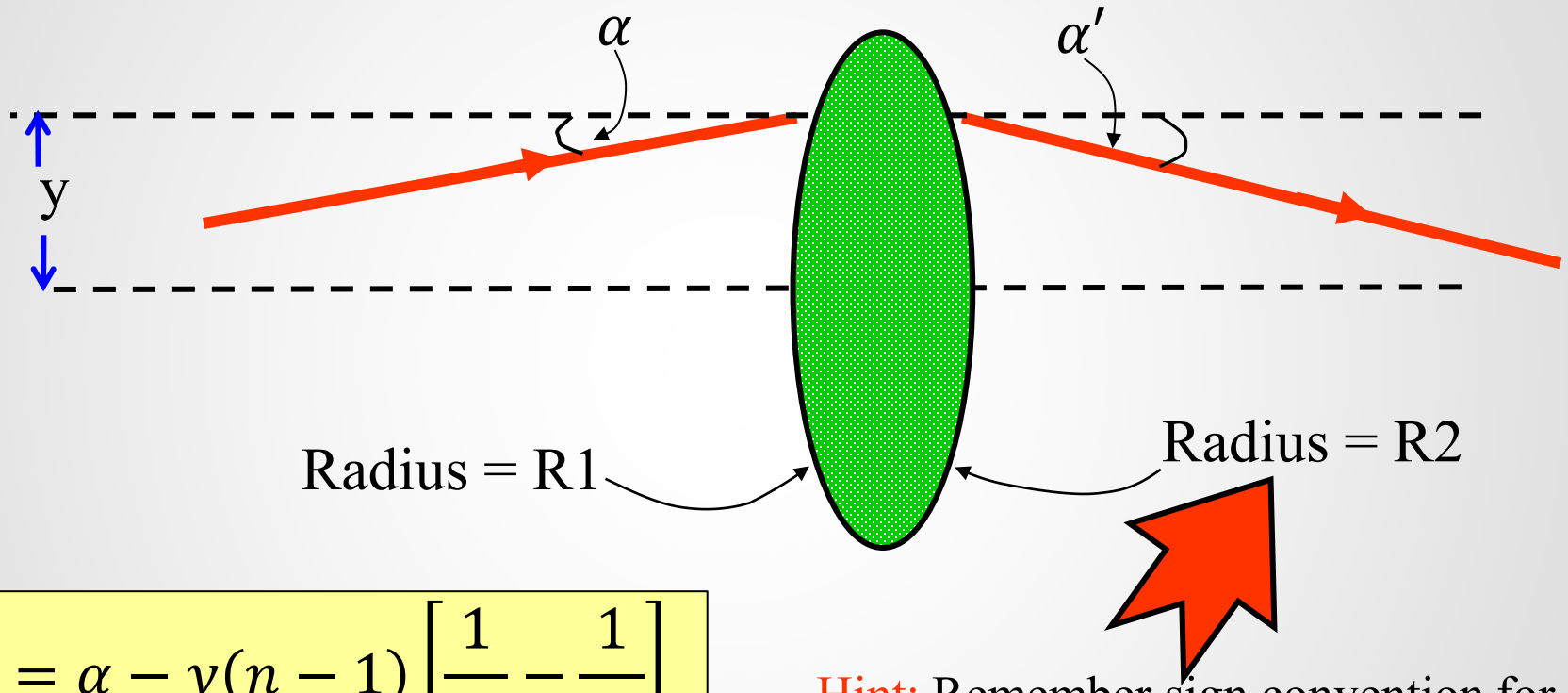
And :

$n_1(\alpha + \beta) = n_2(\alpha' + \beta)$

Refraction at Curved Surfaces

Extension to Multiple Surfaces – (Thin Lenses)

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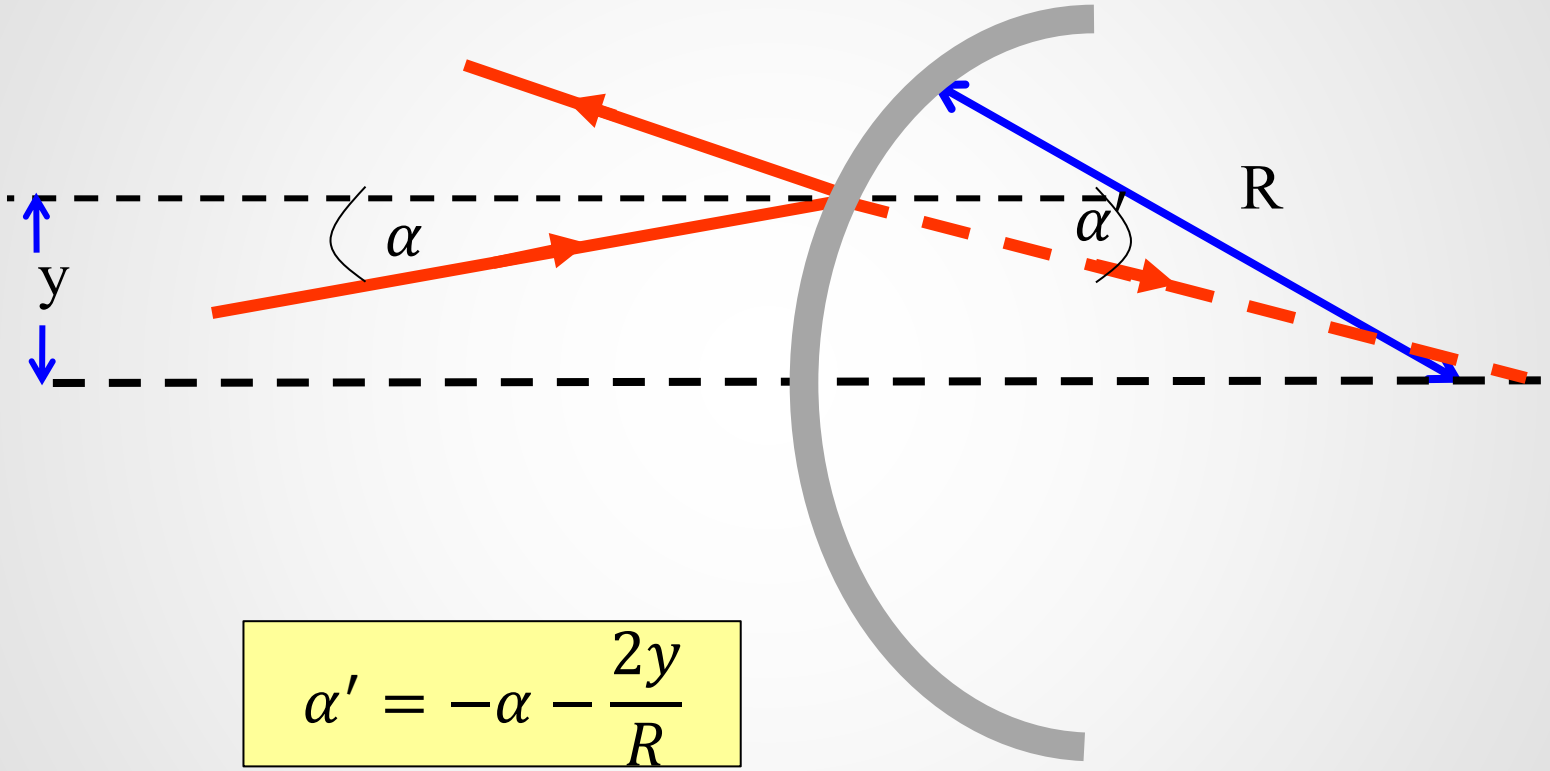


$$\alpha' = \alpha - y(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Hint: Remember sign convention for lens Radii. Positive curvature and centre is to the right (positive z)

Reflection at Spherical Mirror

Paraxial Treatment

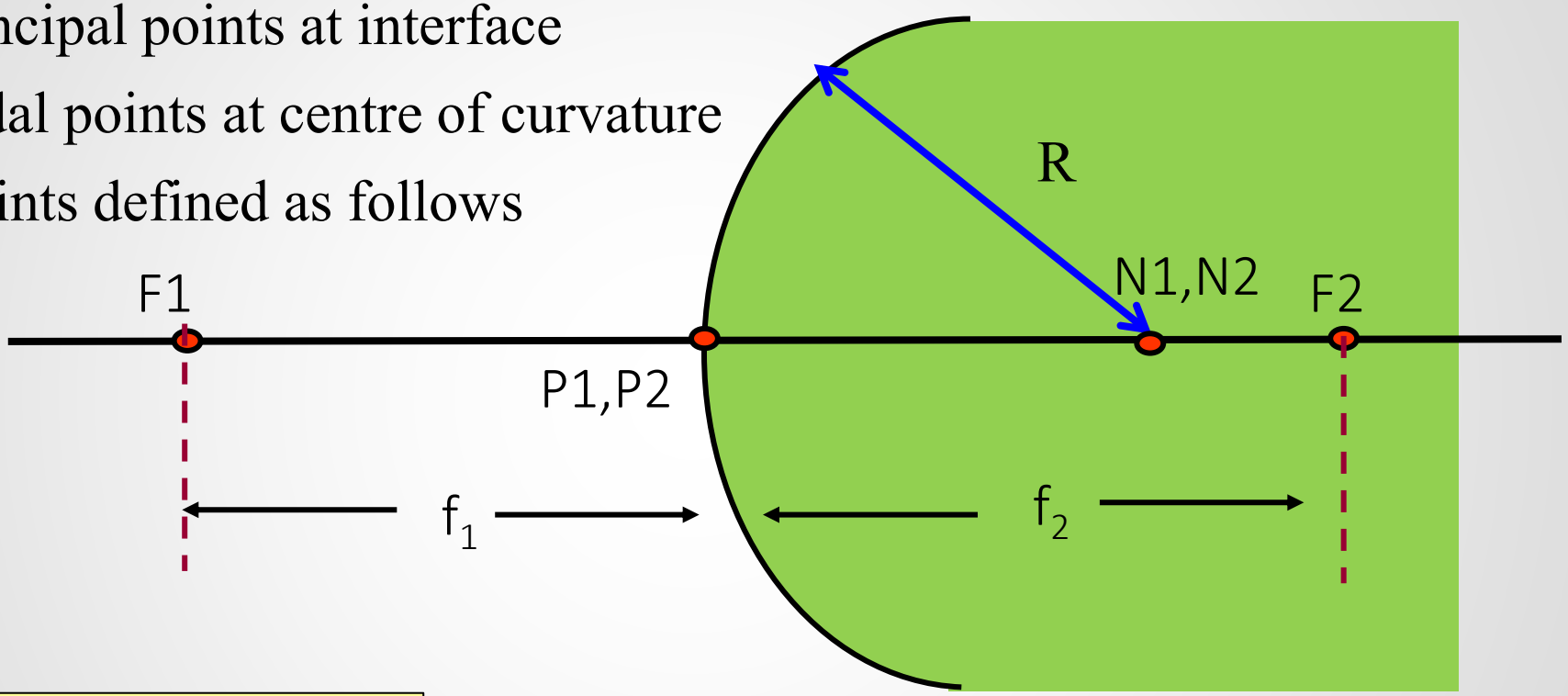


$$\alpha' = -\alpha - \frac{2y}{R}$$

Refraction at Curved Surfaces

Cardinal Points

- Both principal points at interface
- Both nodal points at centre of curvature
- Focal points defined as follows



Using :

$$\alpha' = 0 = \frac{n_1}{n_2}\alpha + \frac{y}{R}\left[\frac{n_1}{n_2} - 1\right]$$

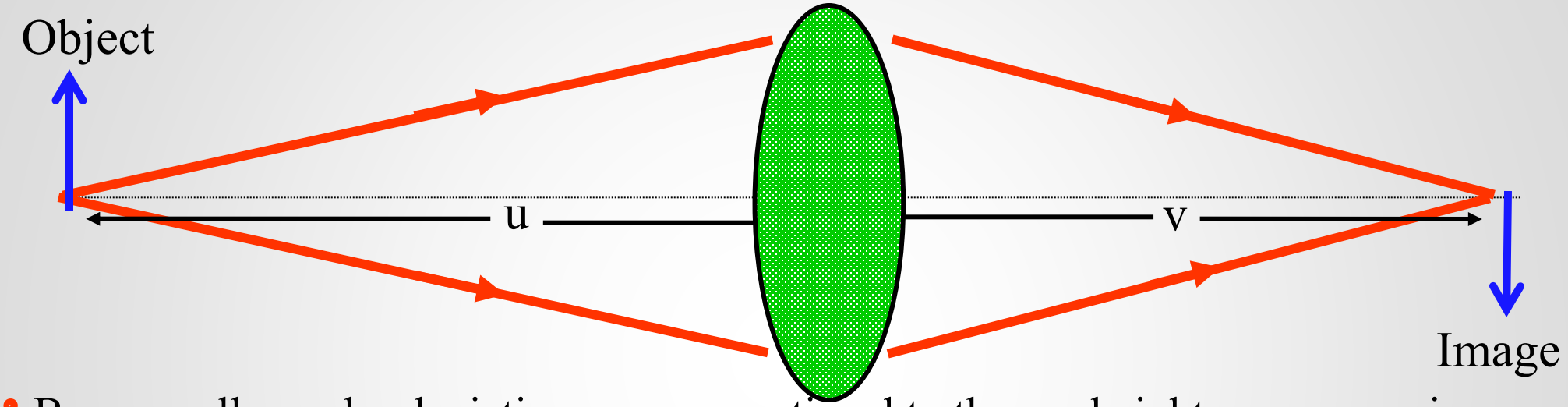
and:

$$y = f_1\alpha$$

$$f_2 = \frac{Rn_2}{n_2 - n_1}$$

$$f_1 = \frac{Rn_1}{n_2 - n_1}$$

Imaging Property of (Thin) Lenses



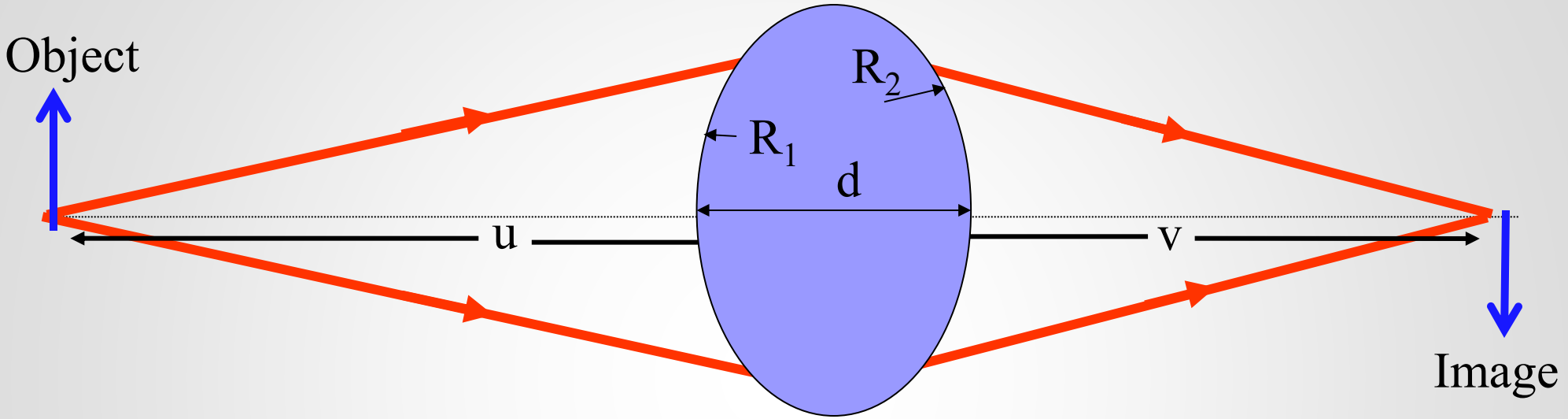
- Because all angular deviations are proportional to the ray height, rays emerging from a single point (object point, u) are always deviated in such a way that they converge to (*or diverge from*) a single point (image point, v).

- ALL RAYS PASSING THROUGH THE CENTRE OF THE LENS ARE UNDEVIATED

$$\frac{1}{u}+\frac{1}{v}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\frac{1}{f}$$

- f is the focal length of the lens

Imaging Property of (Thick) Lenses

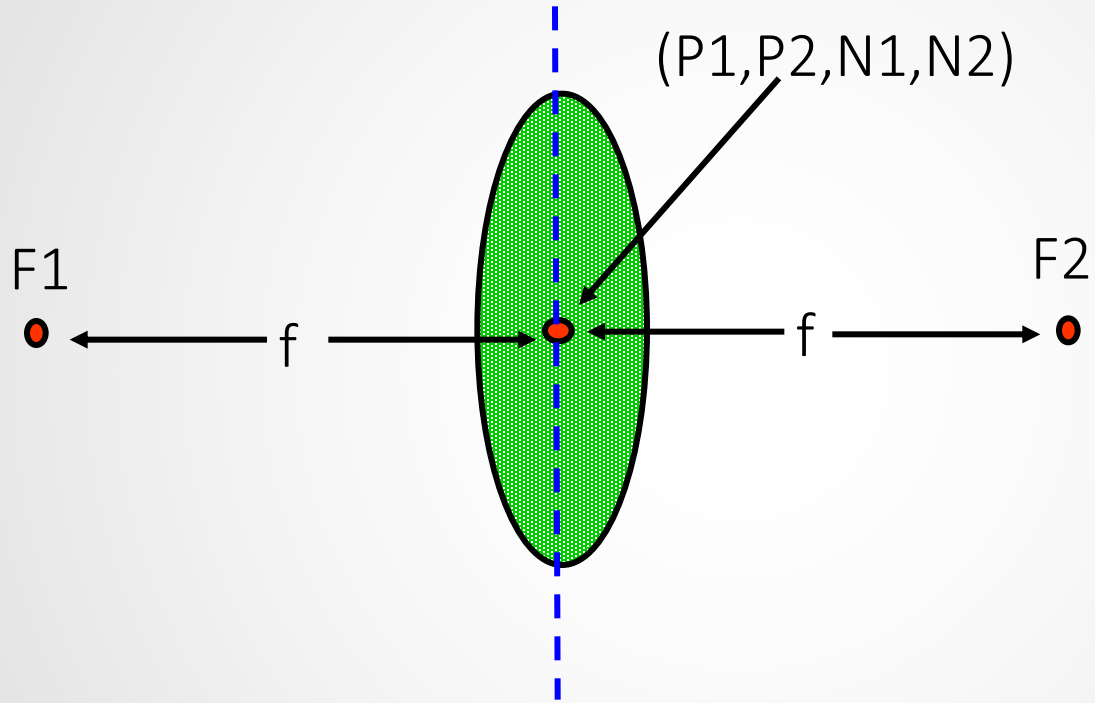


- Lens maker equation

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n-1)d}{nR_1R_2}\right]$$

- Thin lens equation is an approximation with d = 0

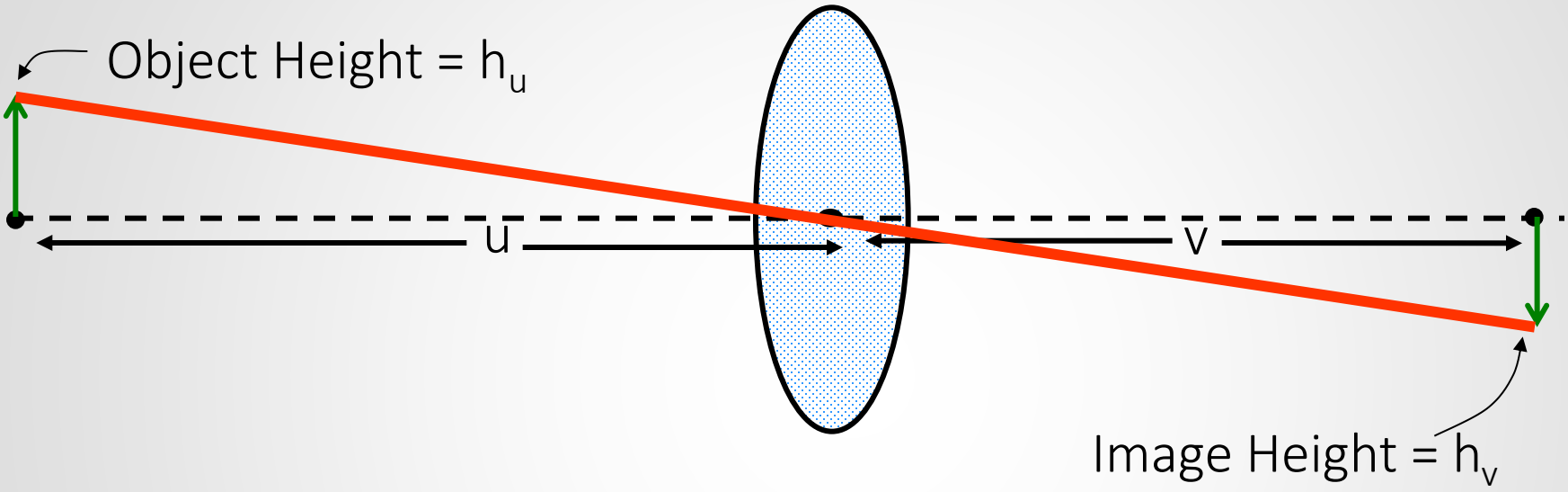
Cardinal Points of a Thin Lens



- Object and Image Space in Same Media

Magnification

Reprise

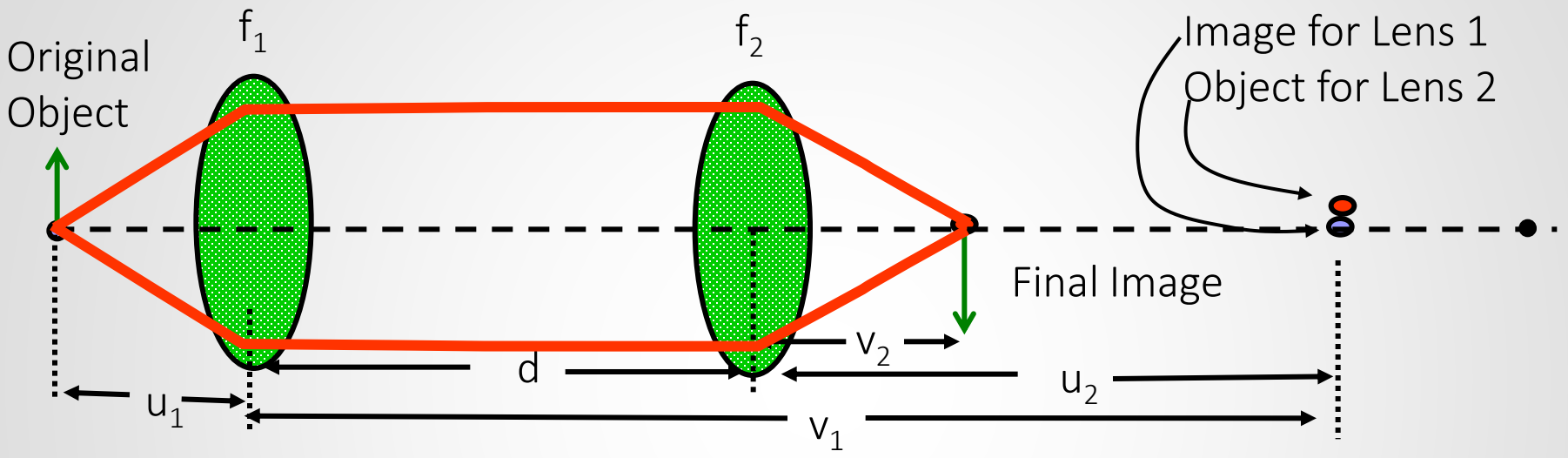


- All rays passing through the centre of the lens are undeviated

$$M_{lateral} = \frac{h_v}{h_u} = -\frac{\mathrm{v}}{\mathrm{u}}$$

$$M_{axial} = \frac{d\mathrm{v}}{d\mathrm{u}} = -M_{lateral}^2$$

Collimating & Focussing Lens Pair



- Combined Magnification: $M = M1 * M2$

$$M1 = -\frac{v_1}{u_1} \qquad M2 = -\frac{v_2}{u_2} = -\frac{v_2}{d - v_1} = \frac{v_2}{v_1 - d} \qquad M = -\frac{v_1}{u_1}\frac{v_2}{v_1 - d}$$

- For collimated beam v1 -> ∞ & u1 = f1; v2 = f2

$$M = -\frac{f_2}{f_1}$$

Geometrical Optics

Limitations

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- **Paraxial Approximation**
  - Only applies for small θ
- **Lens aberrations significant for large θ**
- Ignores wave aspects of light
  - Eikonal analysis assumes changes in ray geometry takes place on scales much larger than wavelength
- **Ray optics relevant only in 'far field'**
  - Away from focussed spot

Geometrical Optics

Summary

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- General description of an ideal imaging system
  - Six Cardinal Points
- Basis of geometrical optics
  - What is meant by a ray
- Fermat's Principle
  - Rays always follows the fastest route
- Reflection and refraction of rays
- Gaussian optics
  - Paraxial approximation
- Paraxial behaviour of optical surfaces