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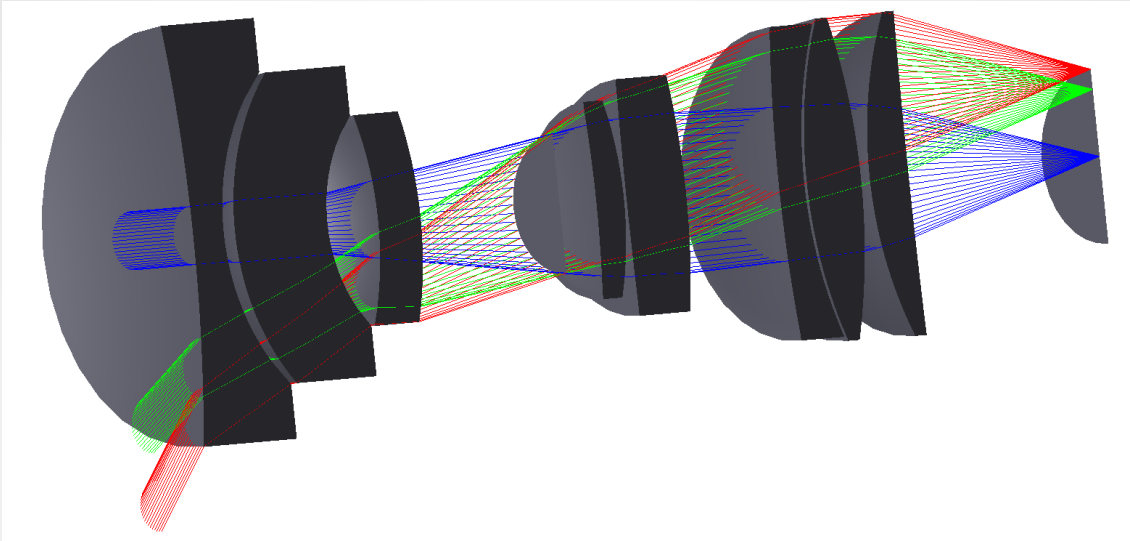

Introduction to Optical Engineering

Michaelmas Term 2026

Lecture 2 – Optical Systems and Aberrations

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Centre for Advanced Instrumentation, University of Durham

Optical Engineering - Outline

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- Geometrical Optics
  - General Theory
  - Fermat's Principle
  - Gaussian Optics and paraxial behaviour of Components and Surfaces
- Optical Systems and Aberrations
  - Matrix Ray Tracing
  - Stops and Pupils definition
  - Introduction to Monochromatic Aberrations (through Spherical aberration example)
- Monochromatic Aberrations
  - Gauss-Seidel Aberrations
  - Behaviour of Lenses and Mirrors – Aplanatic Points
  - Impact of Pupil on Aberrations
- Aspheric Surfaces and Chromatic Aberration
  - Use of Symmetric Aspheric Surfaces
  - Dispersion and Chromatic Aberration
  - Use of Zernike Polynomials in defining OPD or WFE profiles
- Diffraction and Image Quality
  - Diffraction - 'Near Field' and 'Far Field'
  - Gaussian Beam Propagation
  - Definitions of Image Quality



Geometric and Matrix Optics Summary

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- **Gaussian Optics - Reprise**
  - Paraxial approximation
- **Matrix Ray Tracing**
  - Matrices for simple surfaces
- **Matrix ray tracing for complex systems**
  - Derivation of cardinal points
  - Spreadsheet analysis
- **Breakdown of paraxial approximation**
  - Transverse and Longitudinal Aberration
  - OPD and WFE

Paraxial Approximation
Gaussian Imaging

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- Analysis of refraction much easier if we assume
- $\sin\theta << 1$ and $\theta \approx \sin\theta \approx \tan\theta$
- Snell's Law becomes

$$\theta = n\phi$$

- Now easier to formulate Snell's Law for curved surfaces
- Valid for small θ – rays very close to the optical axis
- This approximation leads to perfect image formation
- Can apply previous 'ideal analysis' to real systems
- Approximation breaks down for large angles and leads to imperfections or aberrations

Paraxial Approximation
Restriction of Rays by Aperturing

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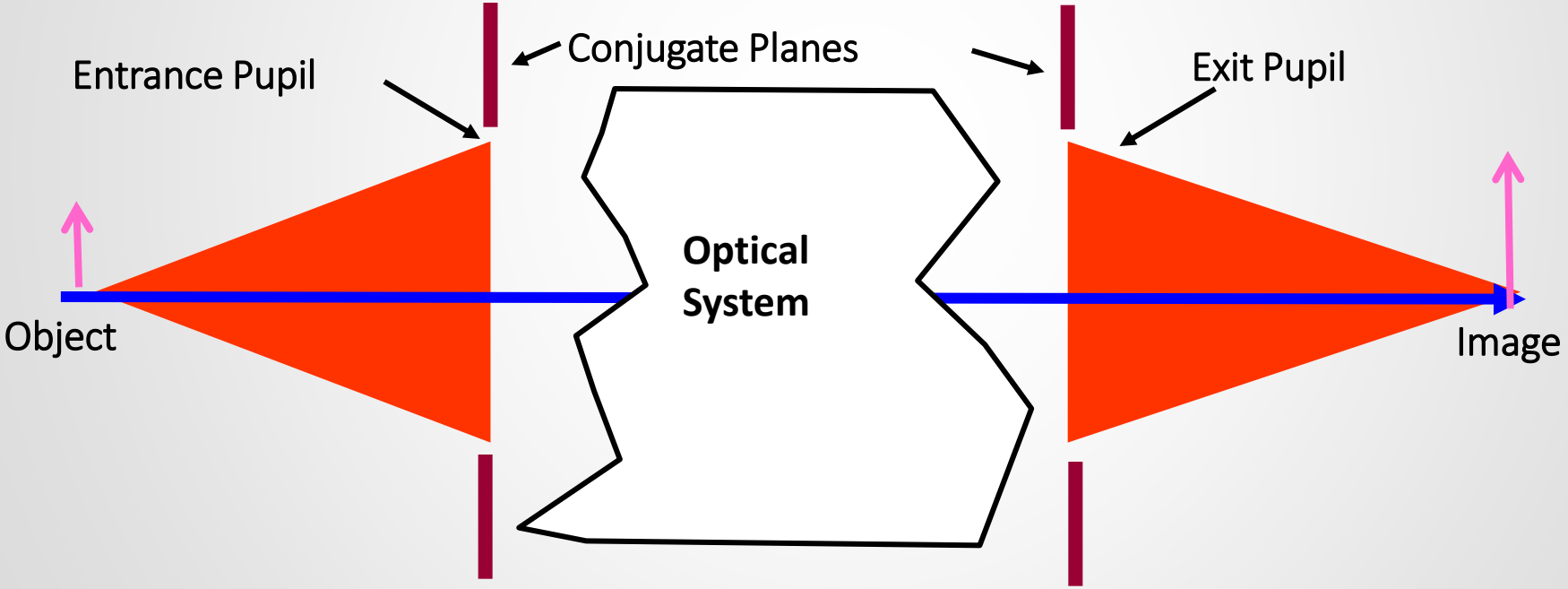
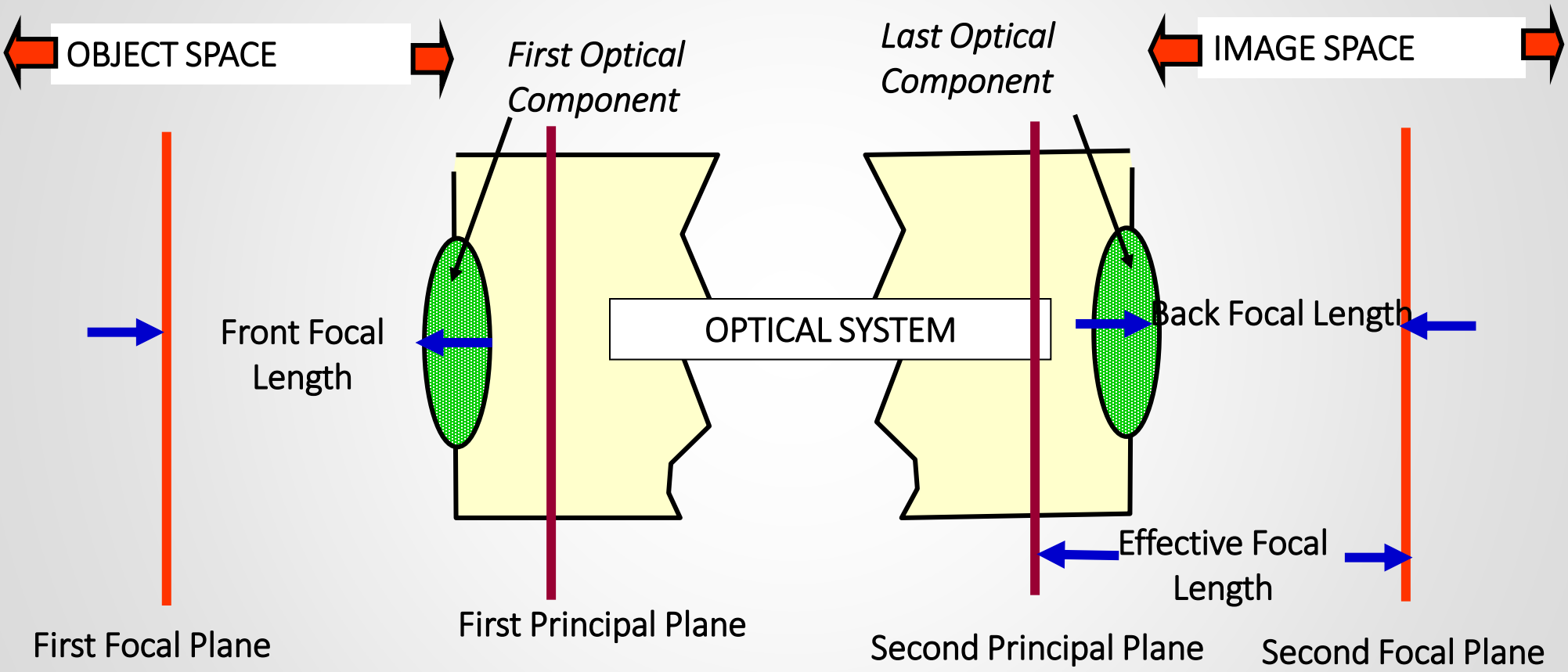
- To restrict rays passing through an optical system to those close to the axis, rays must be rejected by aperturing
  - True of all systems not just ‘paraxial’ or ideal systems

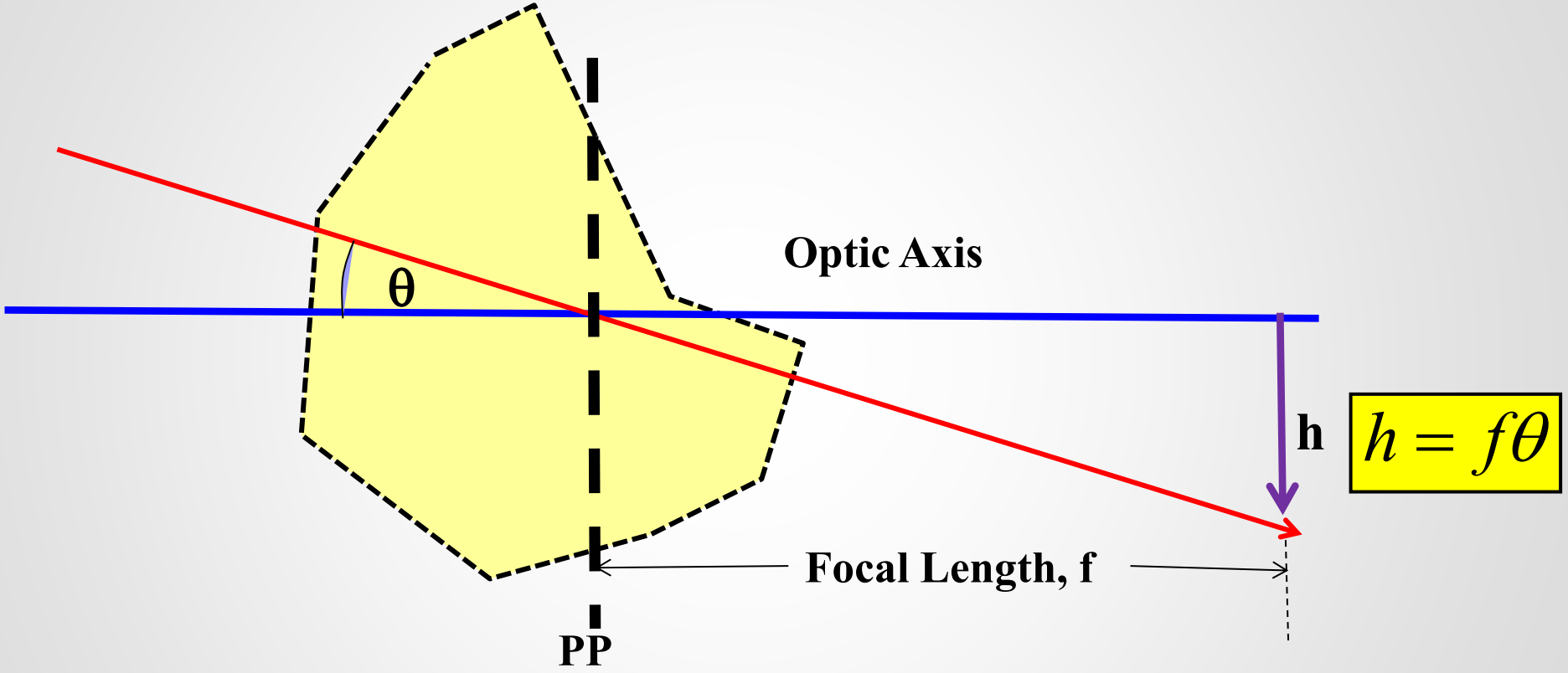


- The entrance pupil is a physical aperture or ‘stop’ imaged in object space

More Definitions



Effective Focal Length
Field Angle and Image Size



- Effective focal length determines image size for given field angle

Paraxial Approximation

Real System Components – Matrix Representation

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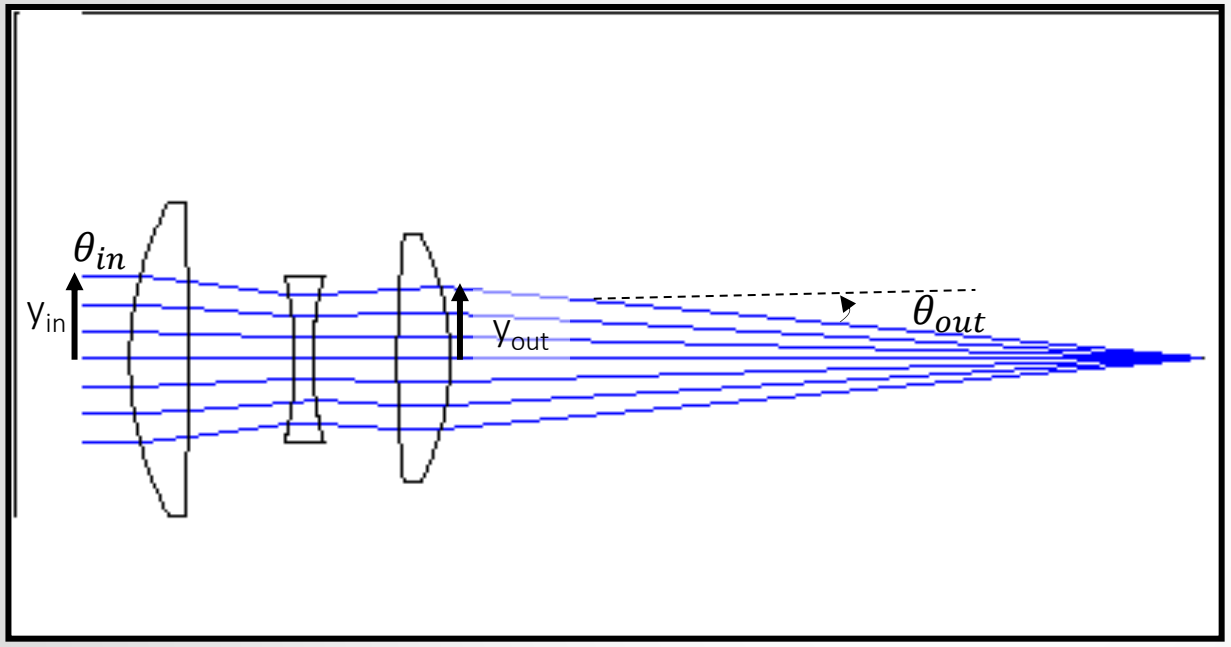
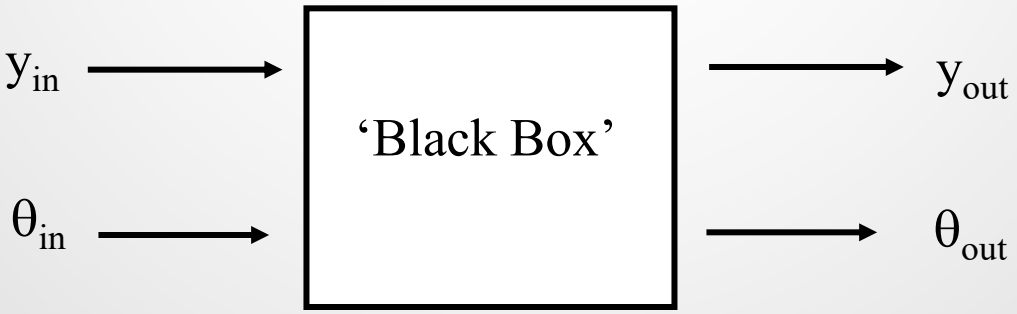
- Using previous analysis we can ‘populate’ ideal system
- Refractive and reflective surfaces
  - Planar and curved surfaces
- Simple lens components
- Use paraxial approximation
- Derive behaviour of system elements using Snell’s Law etc.
- Use Matrix Representation

More Complicated Systems
Matrix Ray Tracing



- What is the effective focal length and where are the principal planes?
- Answer: treat system as 'black box'

- MODEL:



Matrix Ray Tracing

- Basic Formulae

$$y_{out} = Ay_{in} + B\theta_{in}$$

$$\theta_{out} = Cy_{in} + D\theta_{in}$$

- In Matrix Format

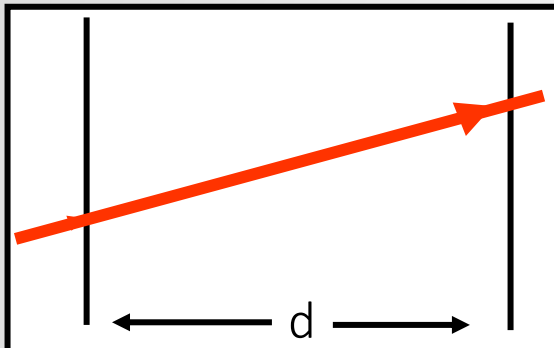
$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

- Treat cascaded components by matrix multiplication
  - $M_{system}=M_N...M_3M_2M_1$
- Positive θ equivalent to positive $\frac{dy}{dz}$

Perfect Optical System Definition

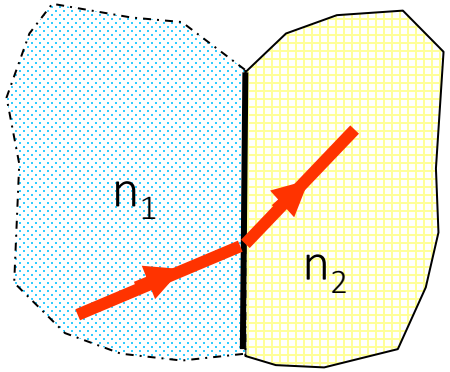
General Ray Tracing

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$



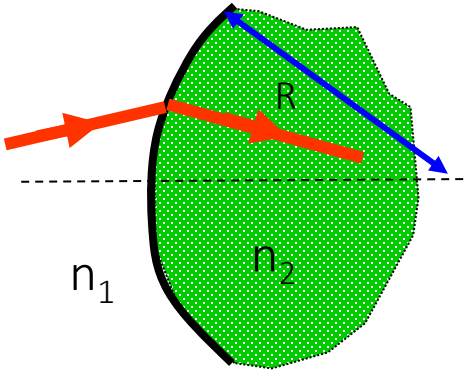
$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Free Space Propagation



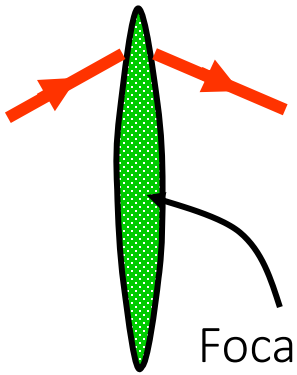
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Refraction at Planar Boundary



$$M = \begin{bmatrix} 1 & 0 \\ \frac{(n_1 - n_2)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

Refraction at a Spherical Boundary

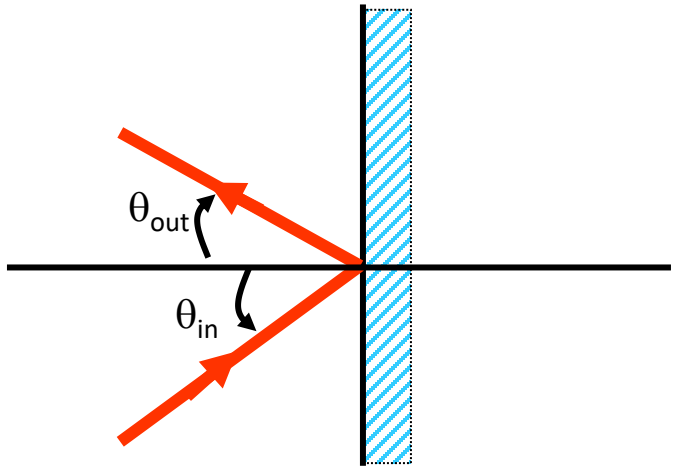


$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Transmission Through a Thin Lens

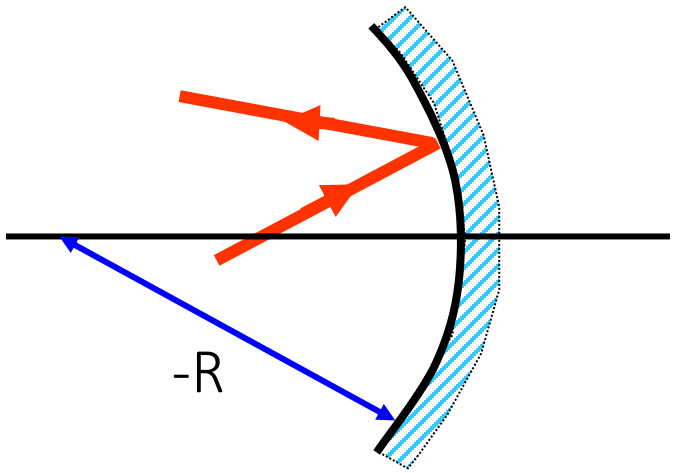
Matrices for Simple Components (2)

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$



$$M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

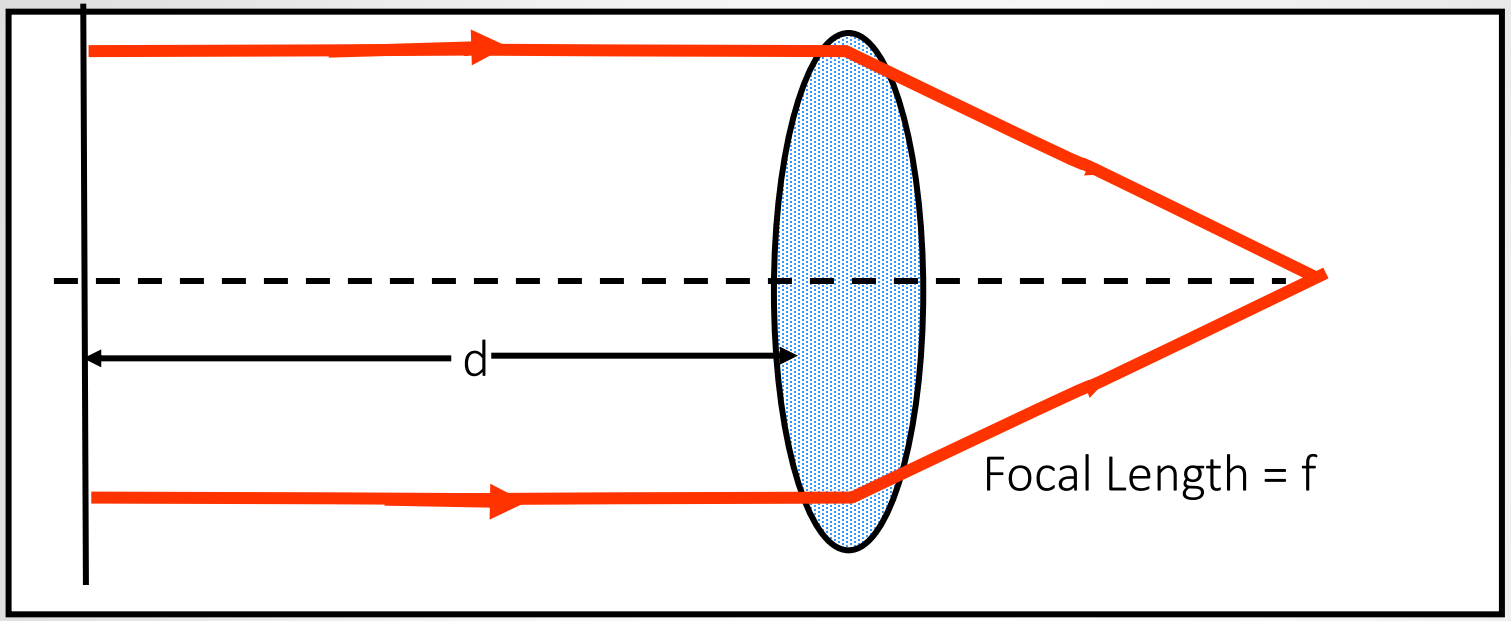
Reflection from Planar Mirror



$$M = \begin{bmatrix} 1 & 0 \\ -\frac{2}{R} & -1 \end{bmatrix}$$

Reflection from Spherical Mirror

Worked Examples
Free Space Propagation Plus Thin Lens



$$M = M_2 M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{bmatrix}$$

- Retrace rays back to start:

$$M = M_1^{-1} M_2 M_1 = \begin{bmatrix} 1 & -d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{bmatrix} = \begin{bmatrix} 1+\frac{d}{f} & \frac{d^2}{f} \\ -\frac{1}{f} & 1-\frac{d}{f} \end{bmatrix}$$

Matrix Ray Tracing
Relation to Ideal Analysis

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- Matrix of Optical System Traced to Origin

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- 1st Focal Point

$$F1 = \frac{D}{C}$$

- 2nd Focal Point

$$F2 = -\frac{A}{C}$$

- 1st Principal Point

$$P1 = B + D\frac{(1-A)}{C}$$

- 2nd Principal Point

$$P2 = \frac{(1-A)}{C}$$

Matrix Ray Tracing
Relation to Ideal Analysis

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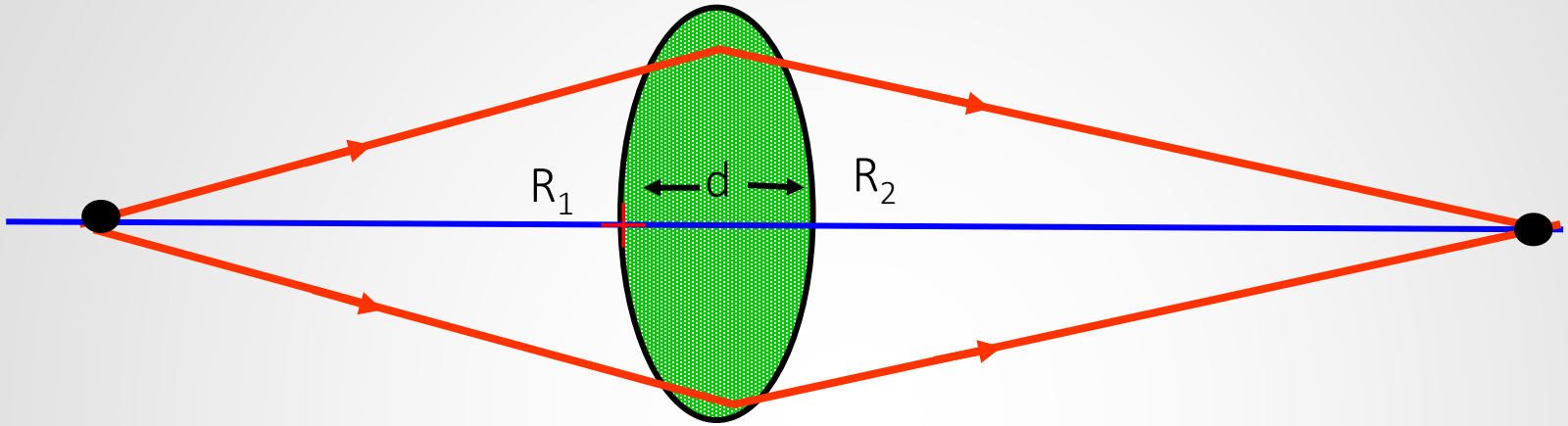
- f1 $$f_1 = -\frac{(AD - BC)}{C} = -\frac{\det(M)}{C}$$
- f2 $$f_2 = -\frac{1}{C}$$
- 1st Nodal Point $$N_1 = \frac{(D-1)}{C}$$
- 2nd Nodal Point $$N_2 = \frac{A(D-1) - BC}{C}$$

- det(M) is the determinant of the Matrix

- n_1 and n_2 are the real indices of object and image spaces – confirming previous suspicions!

Application to Thick Lens (1)



Retrace to Surface

1st Refraction 2nd Surface

Translate to 2nd Surface

Refraction 1st Surface

$$\begin{bmatrix} 1 & -d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{(n-1)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{(1-n)}{nR_1} & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1-\frac{d(n-1)}{R_2} & -dn \\ \frac{(n-1)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1+\frac{d(1-n)}{nR_1} & \frac{d}{n} \\ \frac{(1-n)}{nR_1} & \frac{1}{n} \end{bmatrix}$$

Perfect Optical System Definition

General Ray Tracing

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- Final Matrix

$$\begin{bmatrix} \left[1+d\left((n-1)\left(\frac{1}{R_1}-\frac{1}{nR_1}-\frac{1}{R_2}\right)+\frac{d(n-1)^2}{nR_1R_2}\right)\right] & \left[\frac{d^2(1-n)}{nR_2}\right] \\ \left[-(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)-\frac{d(n-1)^2}{nR_1R_2}\right] & \left[1+\frac{d(n-1)}{nR_2}\right] \end{bmatrix}$$

- Focal Length

$$\frac{1}{f_1}=\frac{1}{f_2}=\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{d(n-1)^2}{nR_1R_2}$$

- Focal Positions

$$F_1=-f\left(1+\frac{d(n-1)}{nR_2}\right)$$

$$F_2=f+d-df\frac{(n-1)}{nR_1}$$

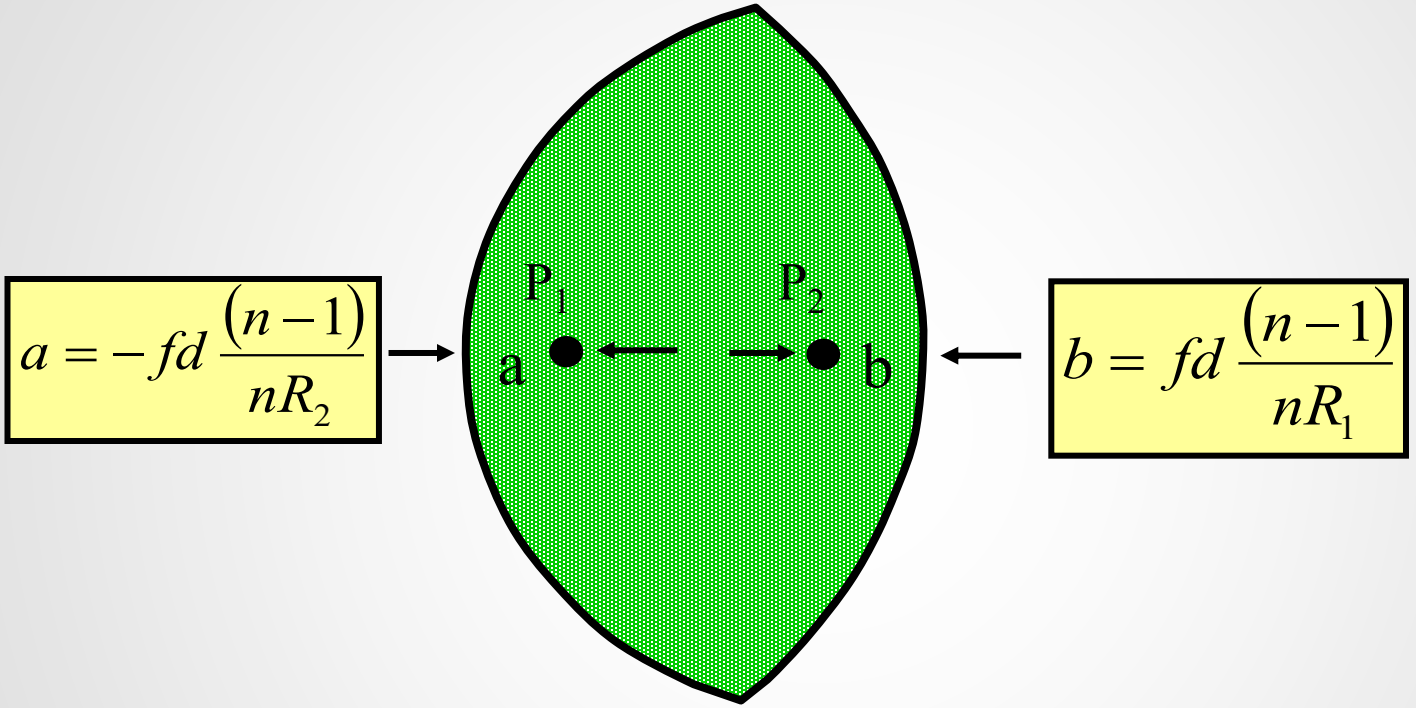
- Principal Planes

$$P_1=-f\frac{d(n-1)}{nR_2}$$

$$P_2=d-df\frac{(n-1)}{nR_1}$$

Nodal Points are Identical

Application to Thick Lens
Principal Points



$$\frac{1}{f_1} = \frac{1}{f_2} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{d(n-1)^2}{nR_1R_2}$$

- Lensmaker's Formula

Matrix Ray Tracing
Spreadsheet Analysis

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- Matrix Ray tracing compatible with spreadsheet analysis
- Complex systems can be analysed for paraxial approximation
- Calculations can be automated
- All relevant system parameters (principal points etc) may be extracted readily

Matrix Optics
Spreadsheet Analysis

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	distance	
	0.3	

n1	n2	R
1	1.87	2.17

Initial Vector	Translate Matrix		New Vector
0	1	0.3	-0.03
-0.1	0	1	-0.1

Curved Surface Matrix		New Vector
1	0	-0.03
-0.2143966	0.534759358	-0.04704404

	distance	
	0.3	

n1	n2	R
1	1.87	2.17

Initial Vector	Translate Matrix		New Vector
0	1	0.3	0.06
0.2	0	1	0.2

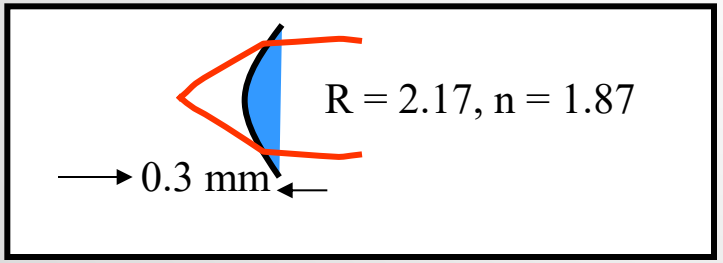
Curved Surface Matrix		New Vector
1	0	0.06
-0.2143966	0.534759358	0.094088075

	distance	
	0.3	

n1	n2	R
1	1.87	2.17

Initial Vector	Translate Matrix		New Vector
0	1	0.3	-0.06
-0.2	0	1	-0.2

Curved Surface Matrix		New Vector
1	0	-0.06
-0.2143966	0.534759358	-0.09408808



Matrix Ray Tracing
Summary

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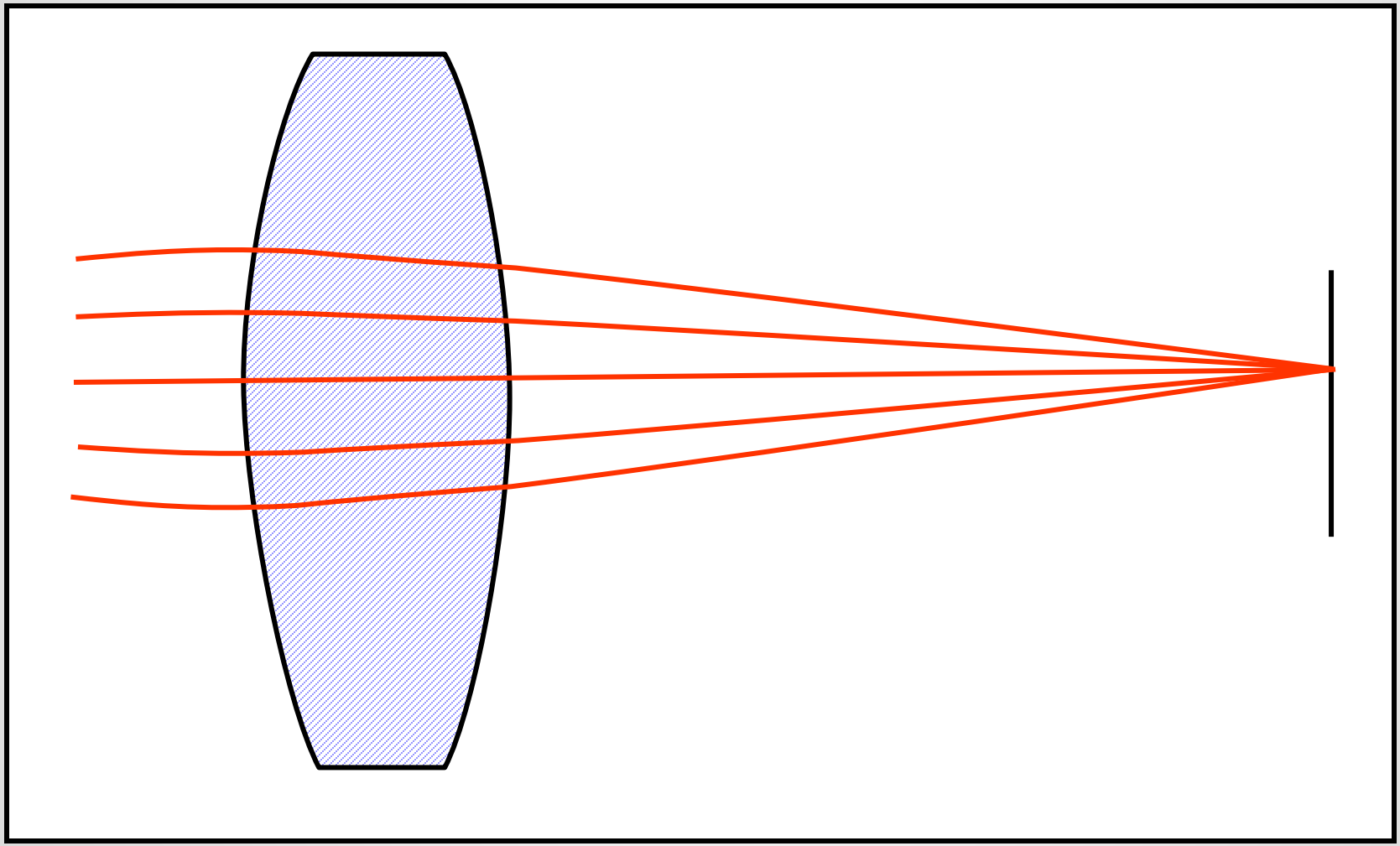
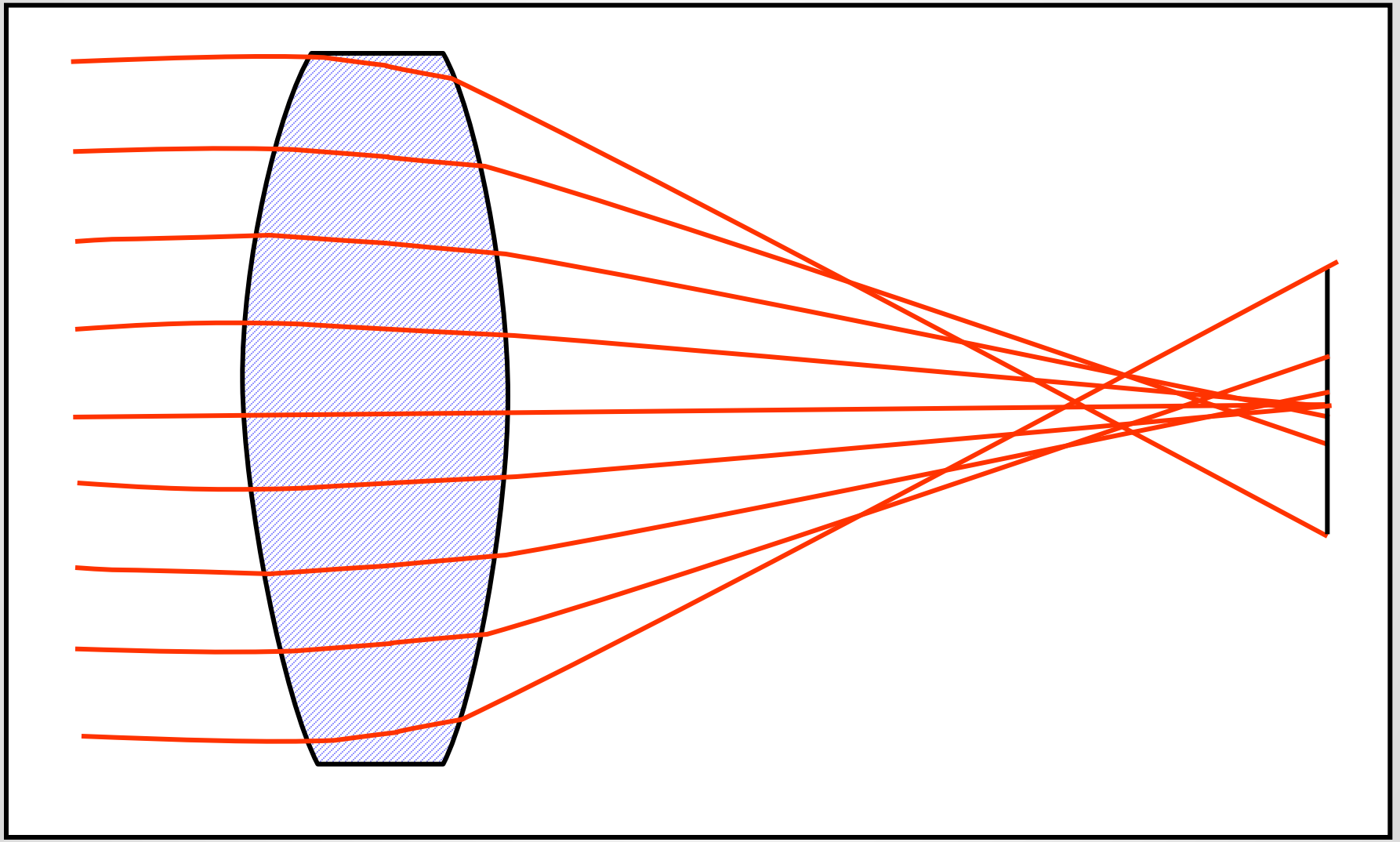
- Useful tool for basic ray tracing
- Simple optical surfaces and components catered for
- Matrix multiplication allows treatment of more complex systems
- Can be easily integrated into spreadsheets
- Use to ‘sketch out’ basic system at beginning of design process
- Helps establish first order parameters, such as effective focal length, component positions etc, at outset of design
- More complex optimisation can follow

Ideal Optical System
Paraxial Approximation



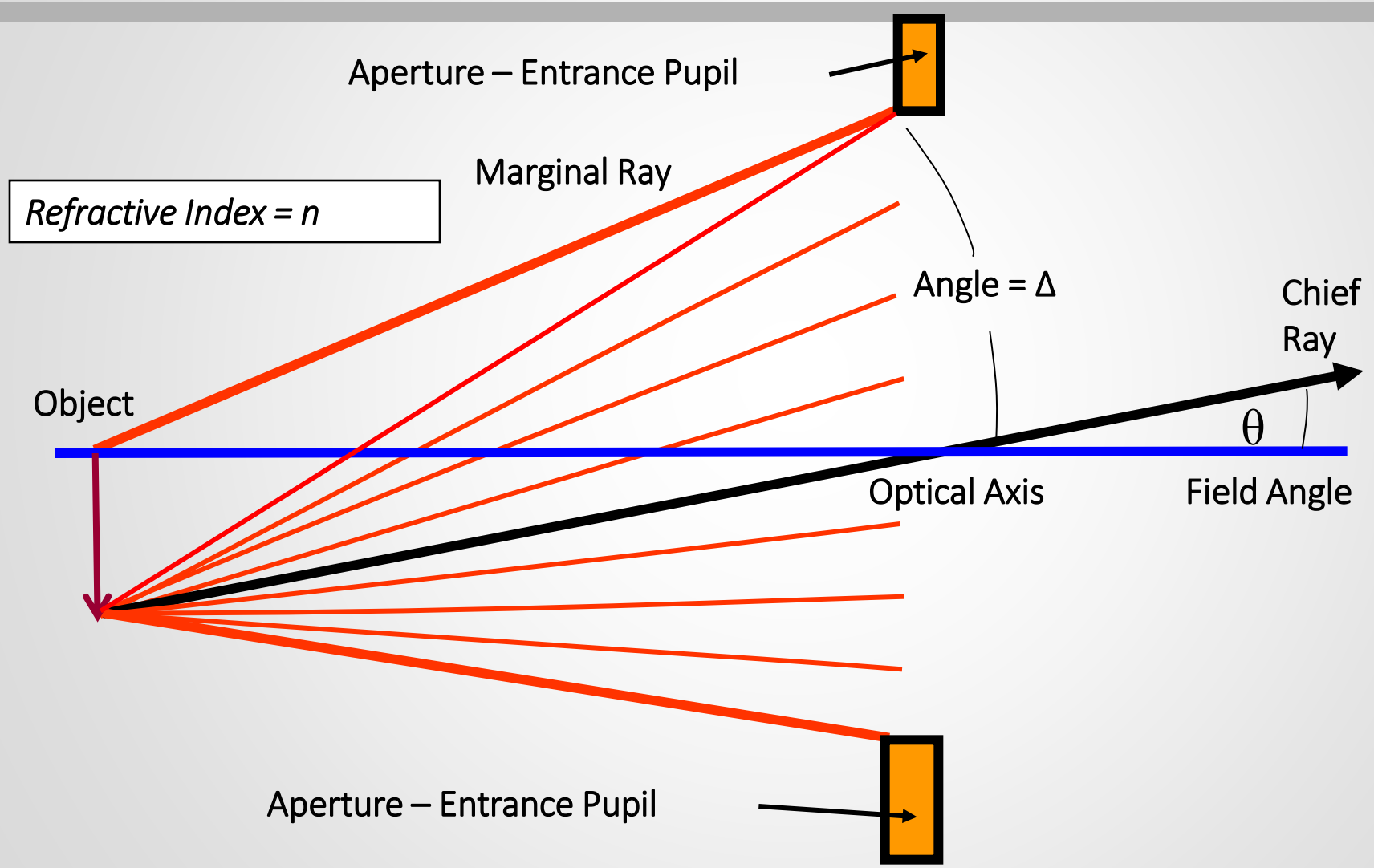
Aberration
Breakdown of Paraxial Approximation



Aberration

- Aberration where paraxial approximation breaks down
  - $\sin\theta \neq \theta$
- Error terms of order θ^3
  - Increase aperture by factor of 2 transverse aberration increased factor of 8
- Very strong dependence on lens aperture
- Small apertures – low aberration
  - Paraxial approximation may apply as working approximation
- Wide aperture lenses – large aberration

Aberration
Some Definitions - Pupil



Entrance and Exit Pupils

More Complex Systems

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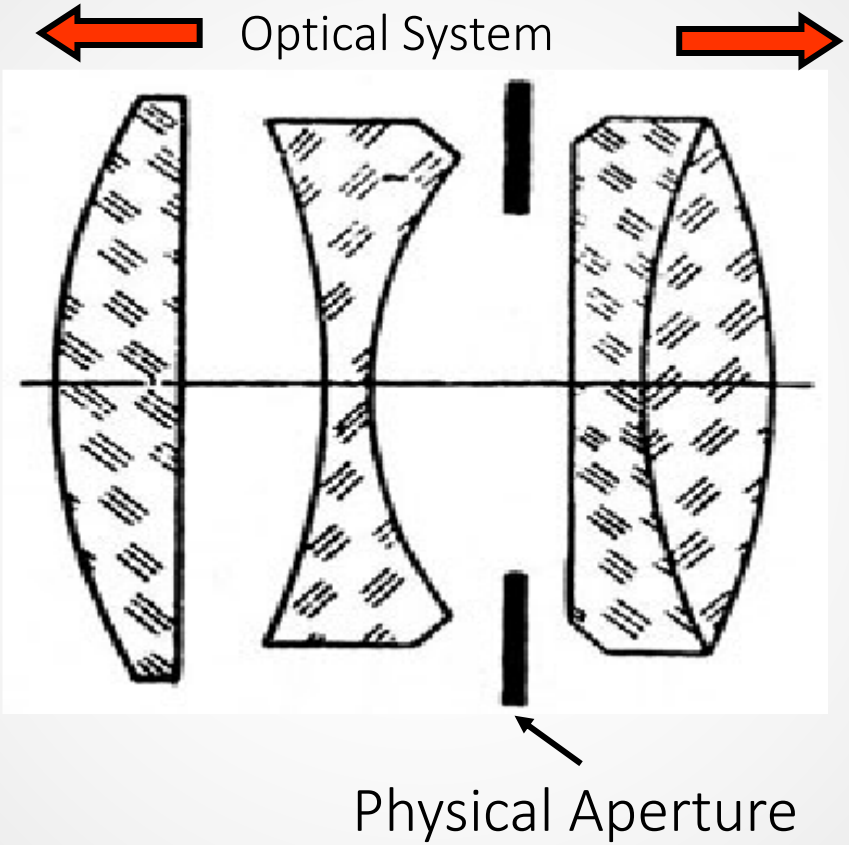

- Location of entrance and exit pupils uses the paraxial approximation, e.g. using matrix ray tracing

Aberrations

More Definitions

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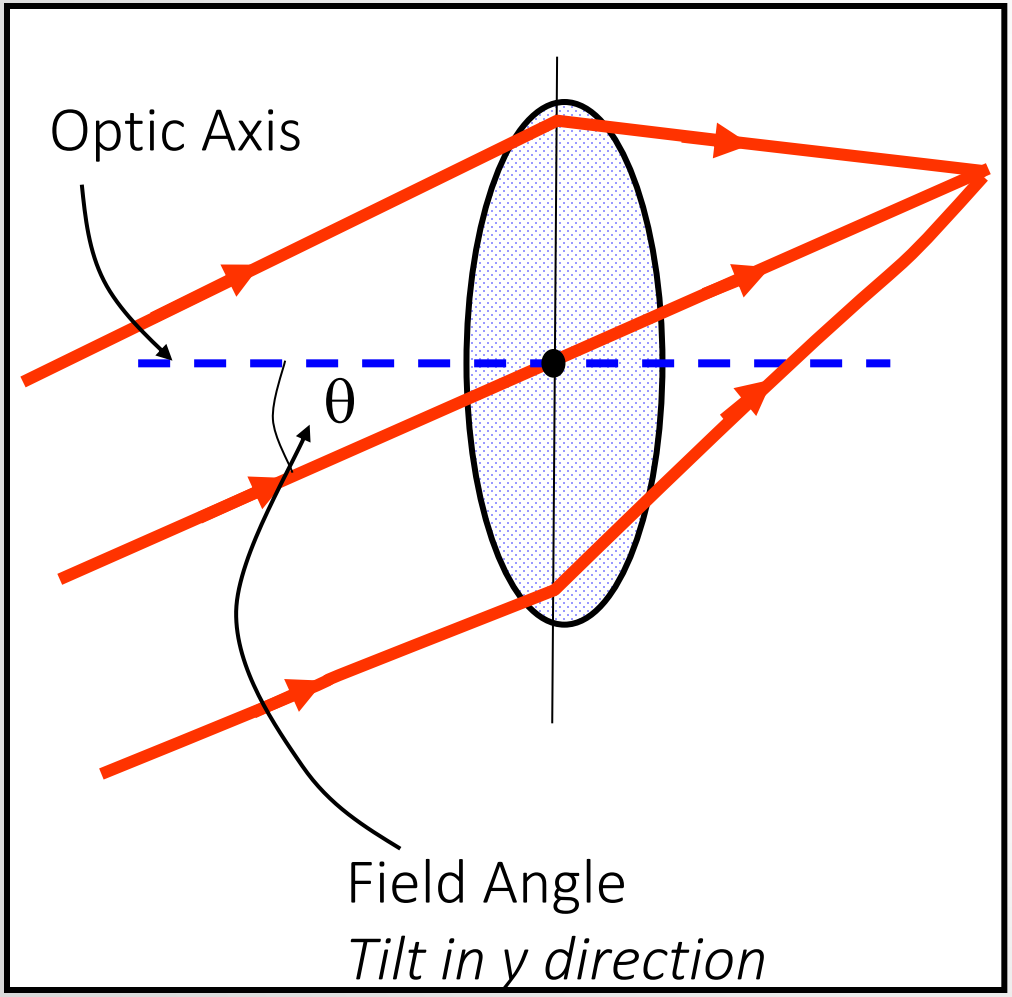
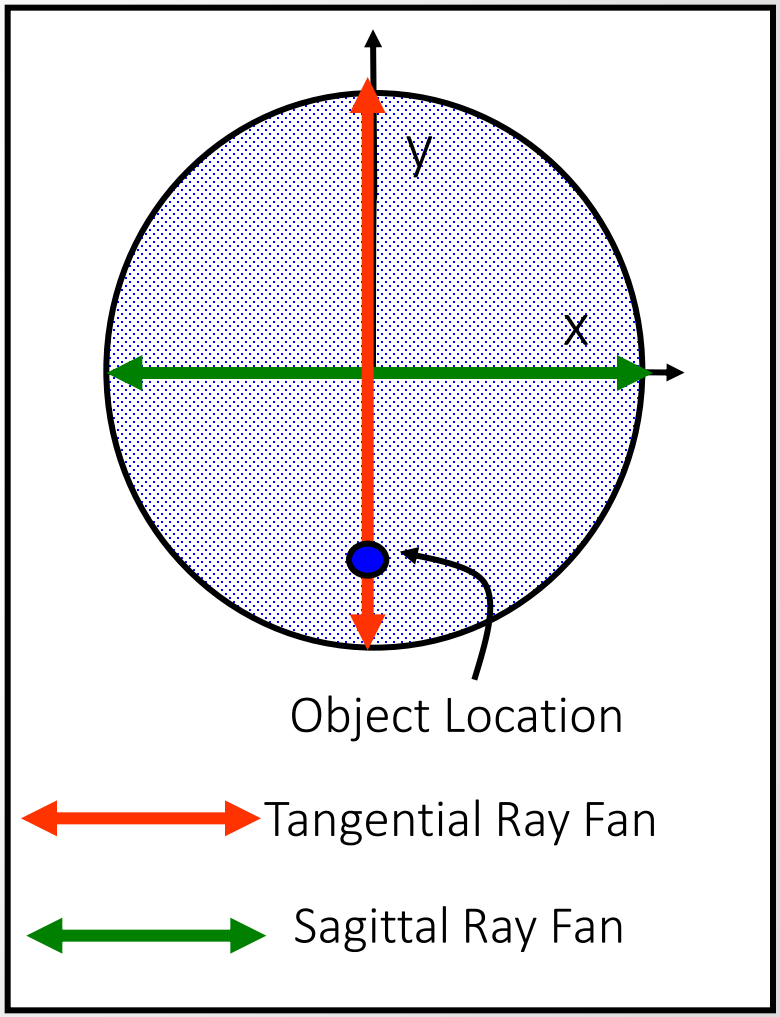
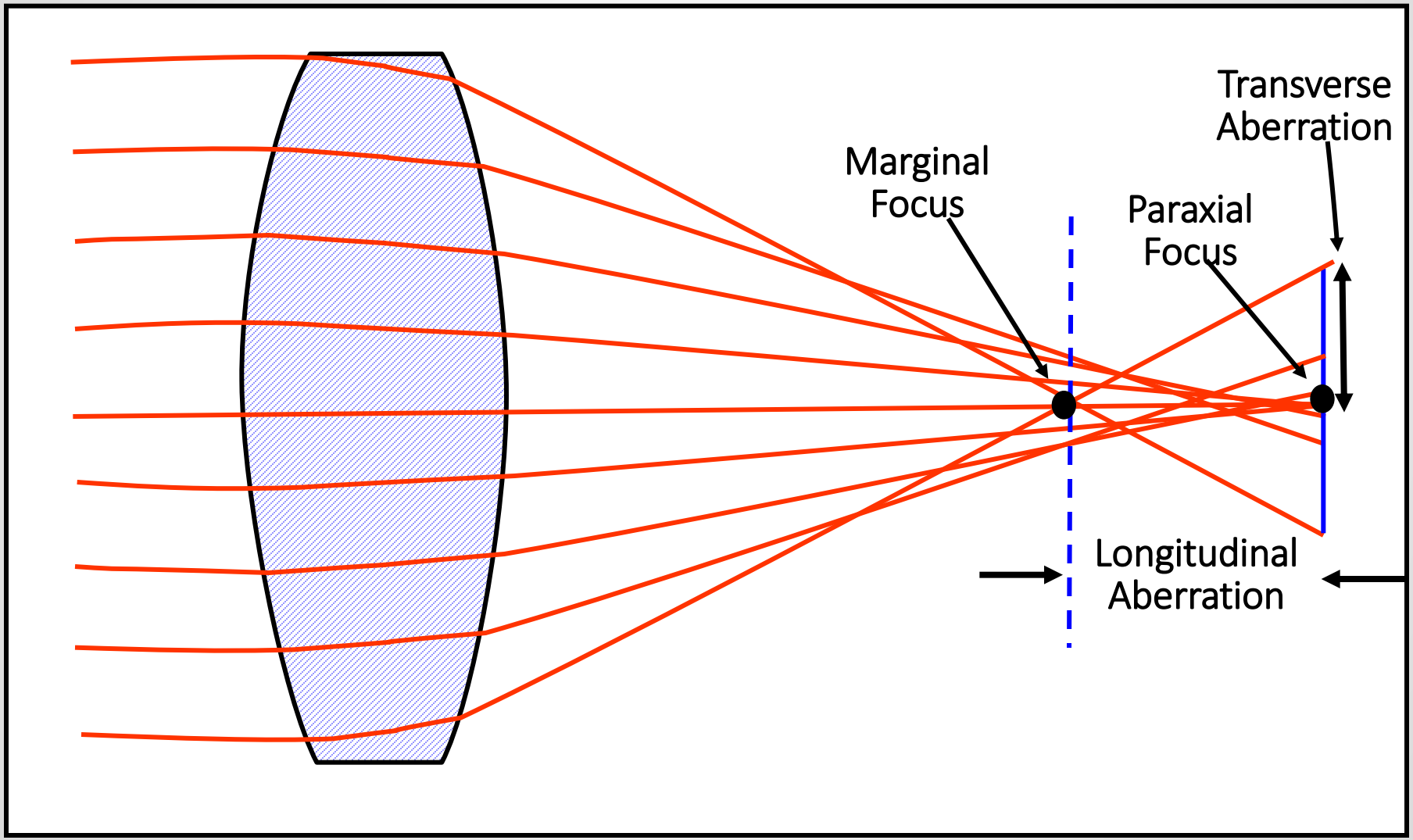
(Side View)



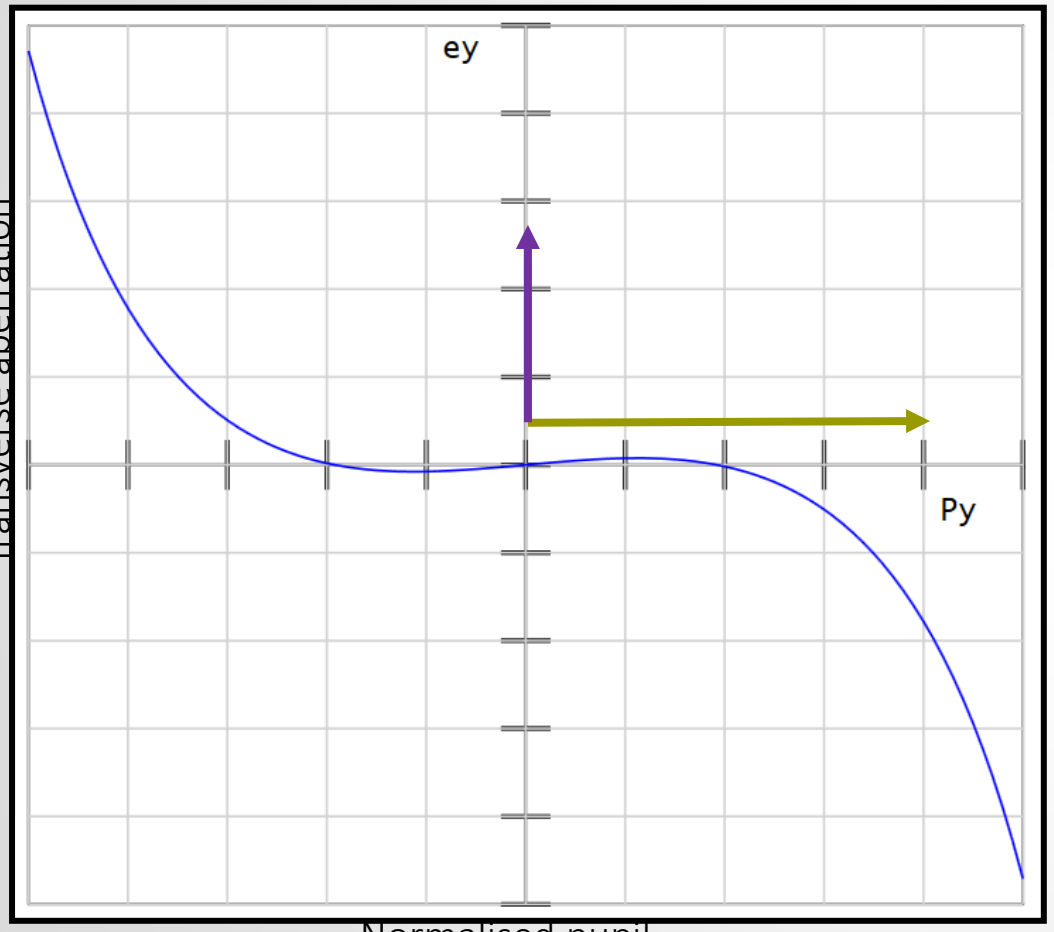
(View Along Optic Axis)

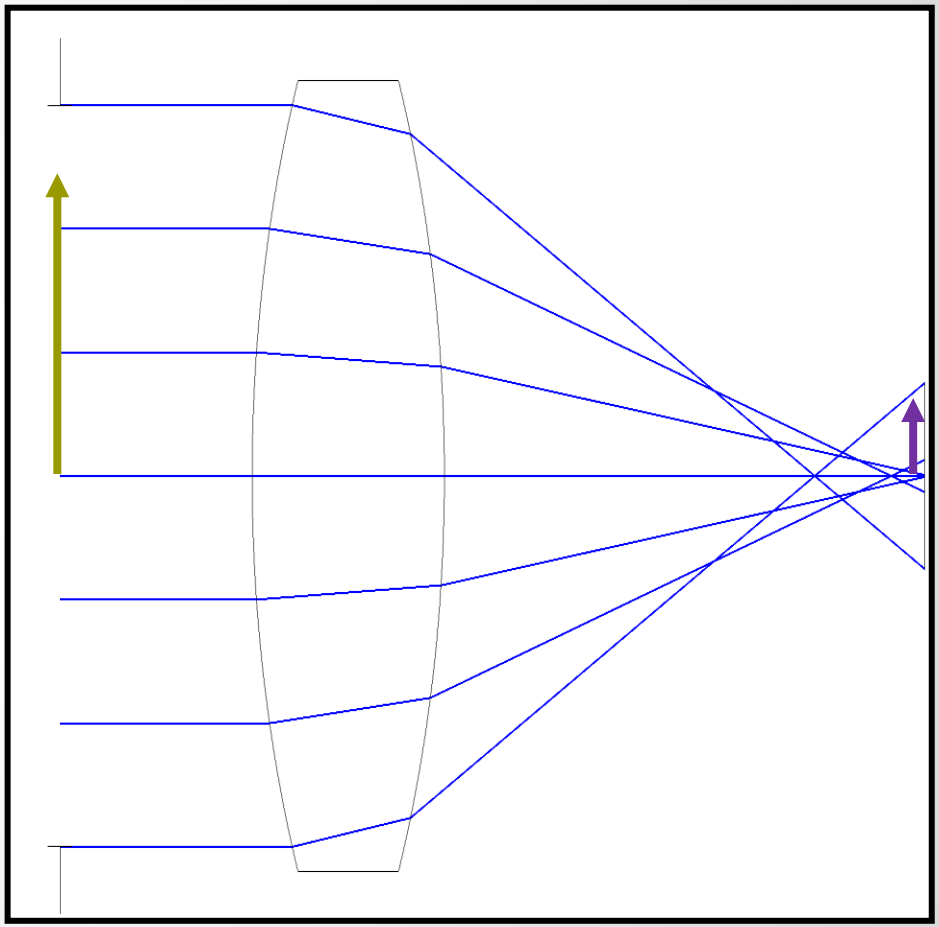


Aberration
Transverse vs Longitudinal



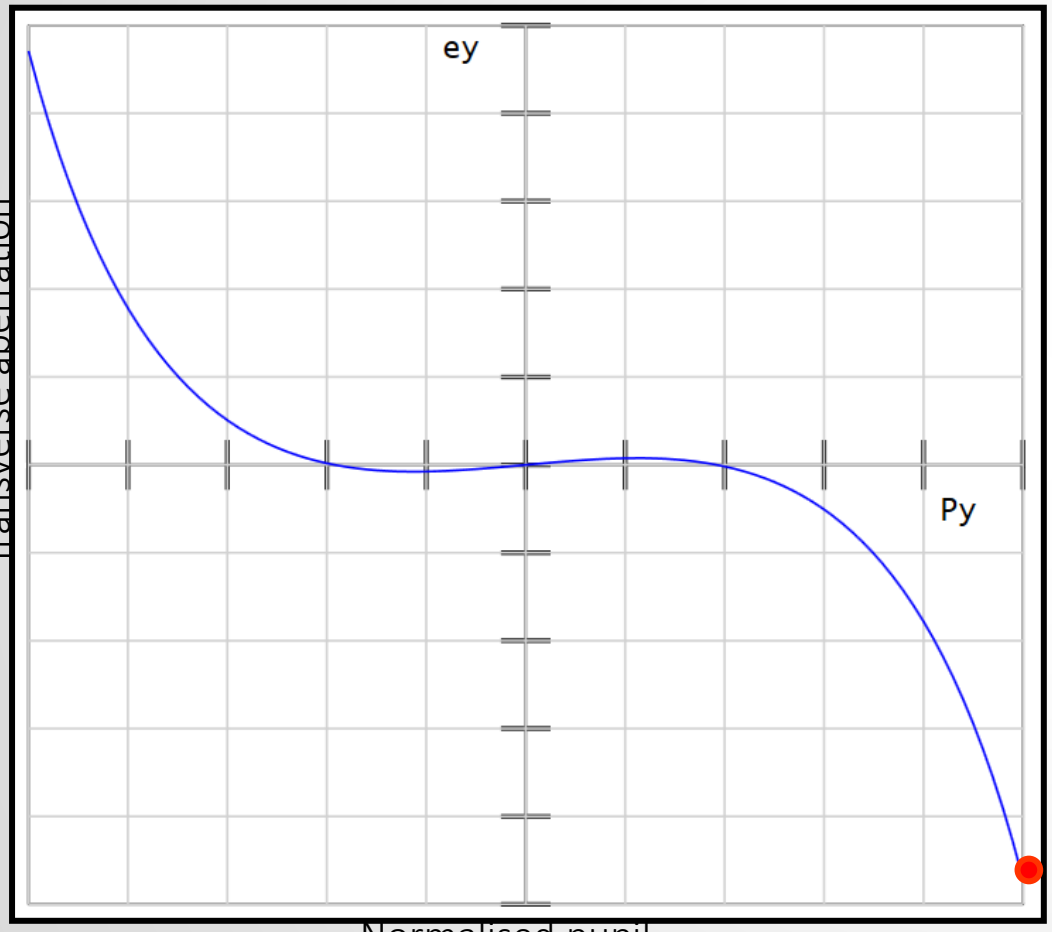
Ray Fan

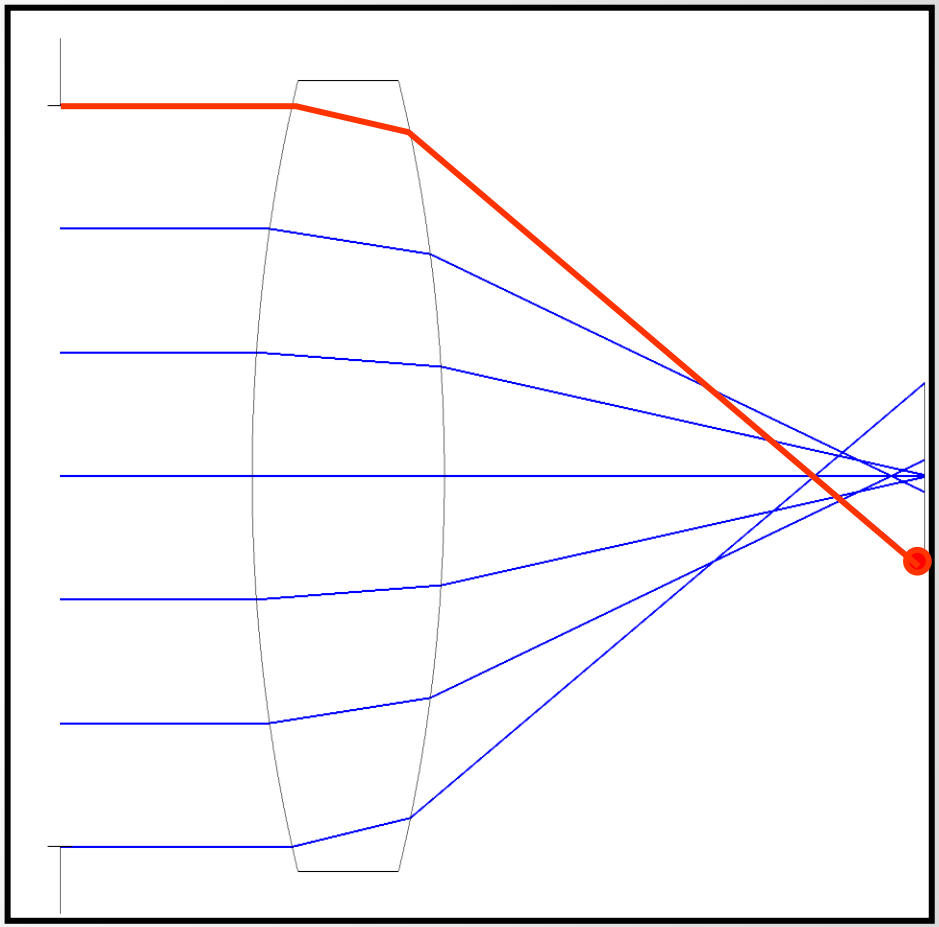




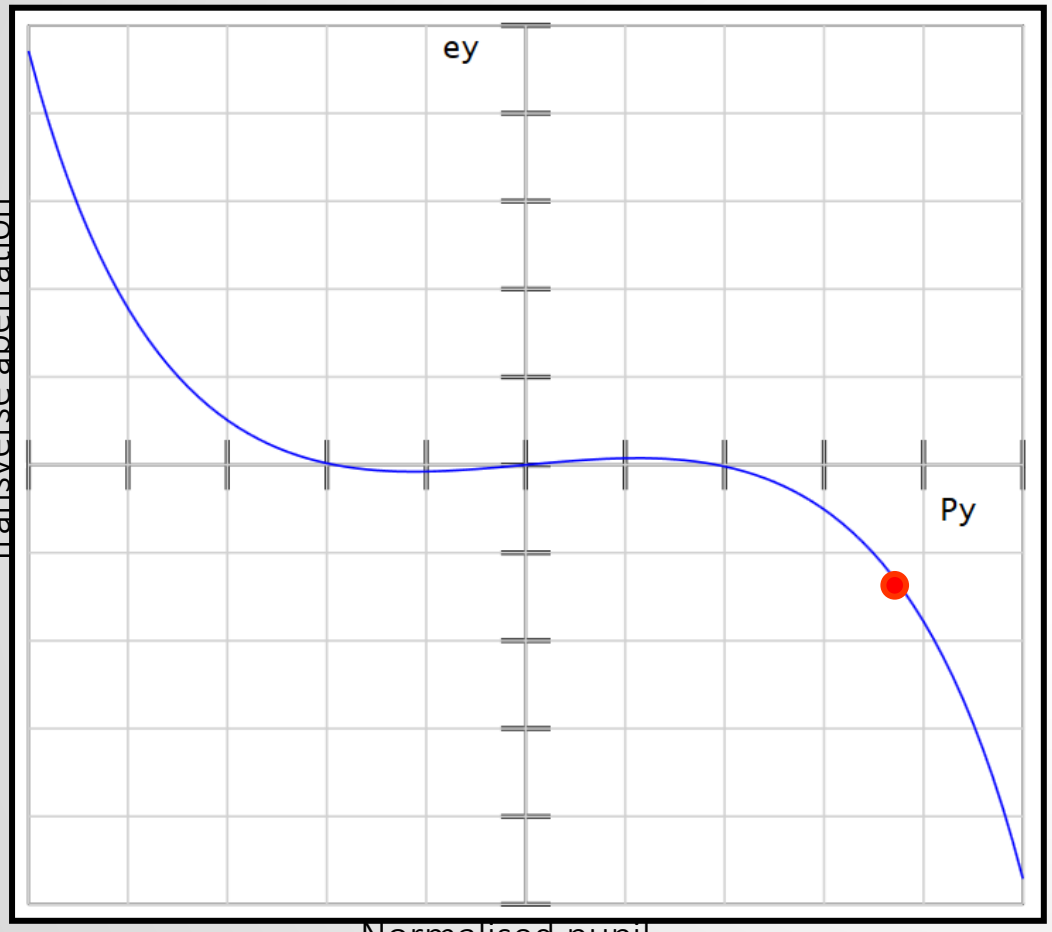
- A ray fan is a plot of transverse aberration against normalised pupil, P

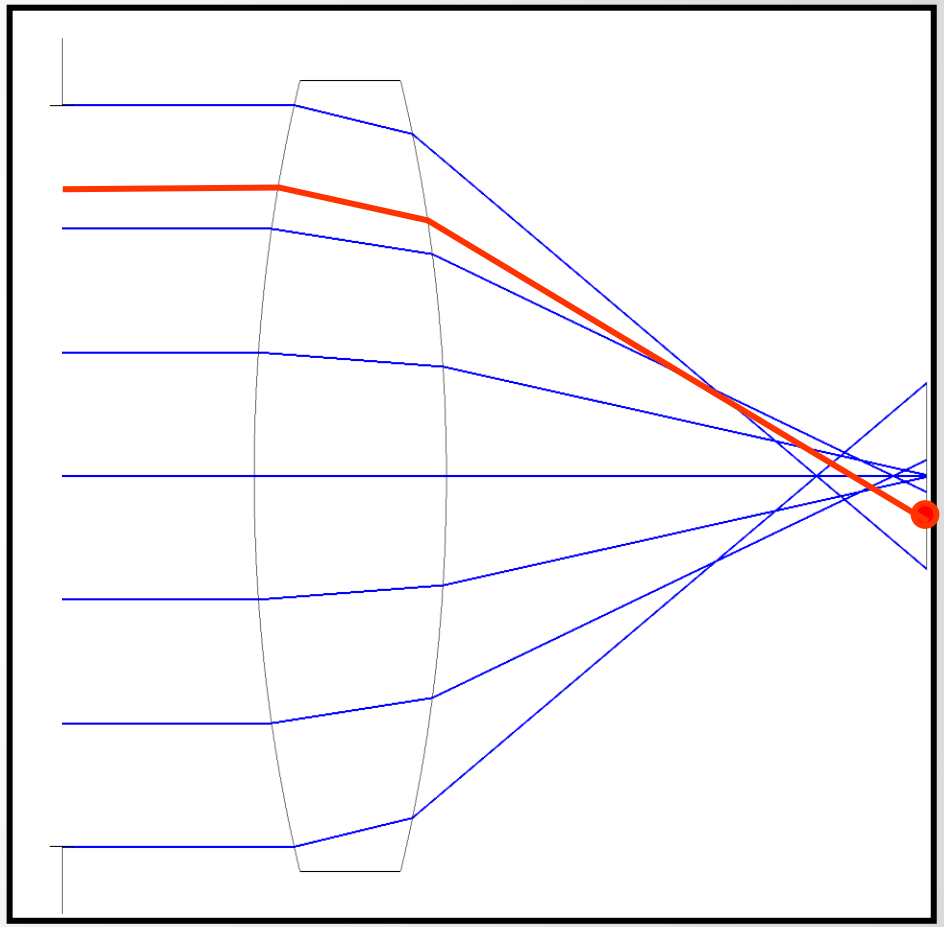
Ray Fan



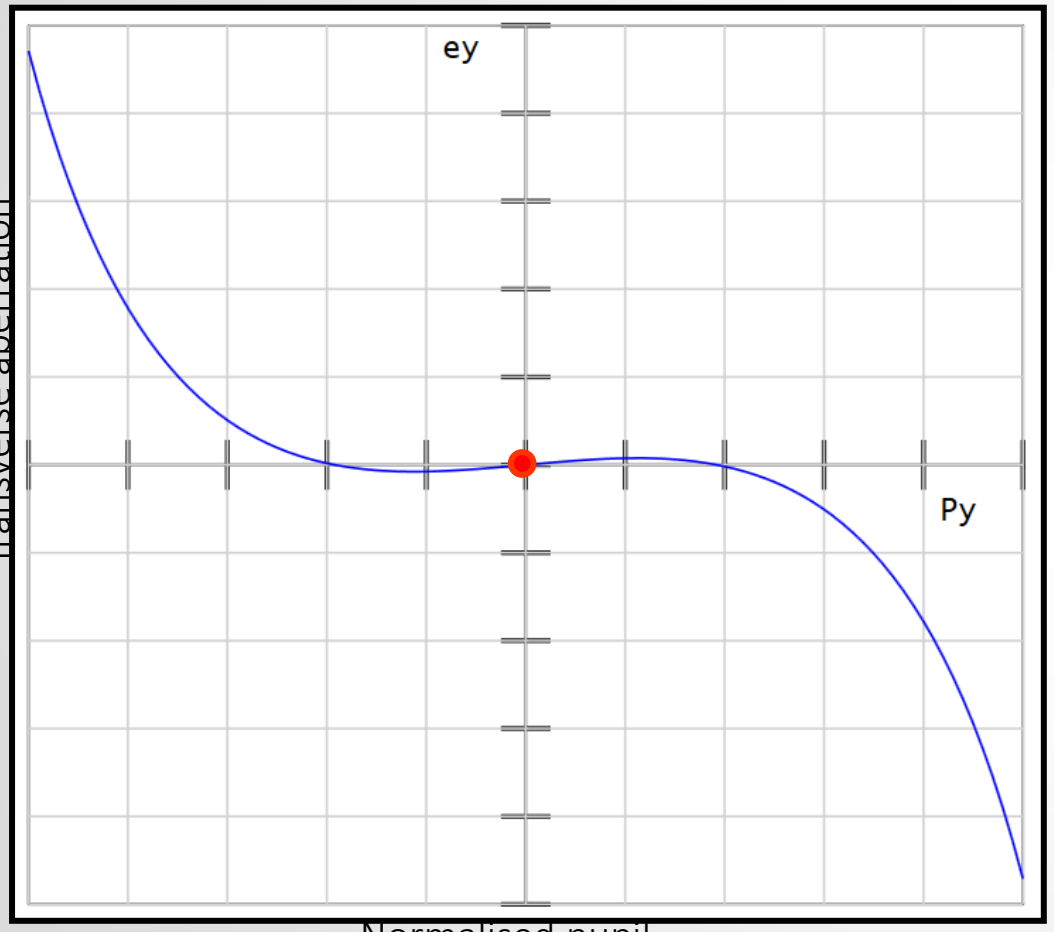


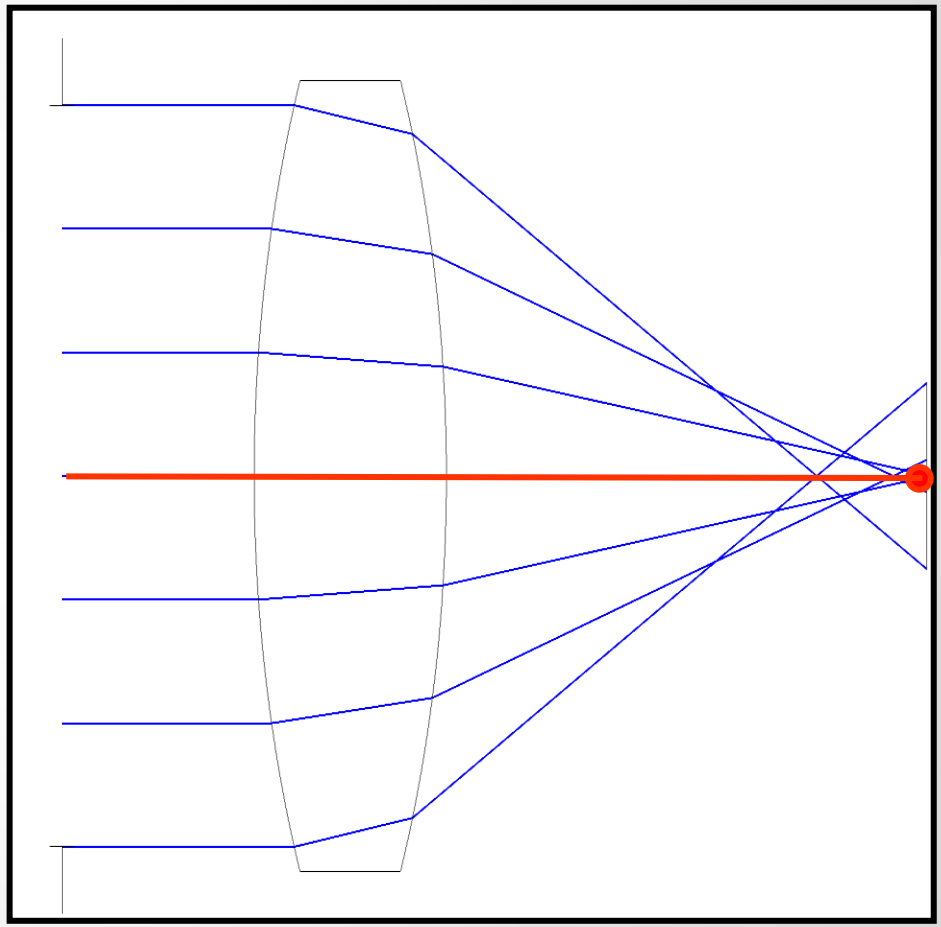
Ray Fan



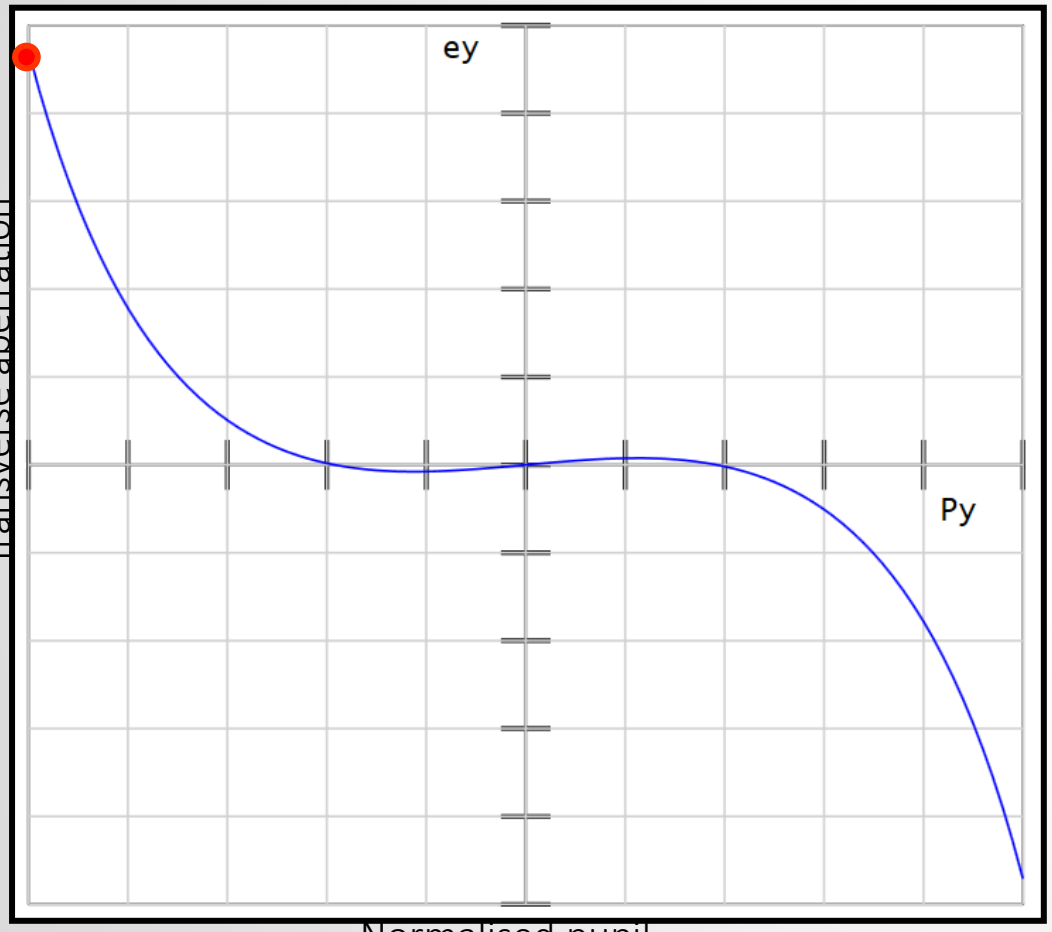


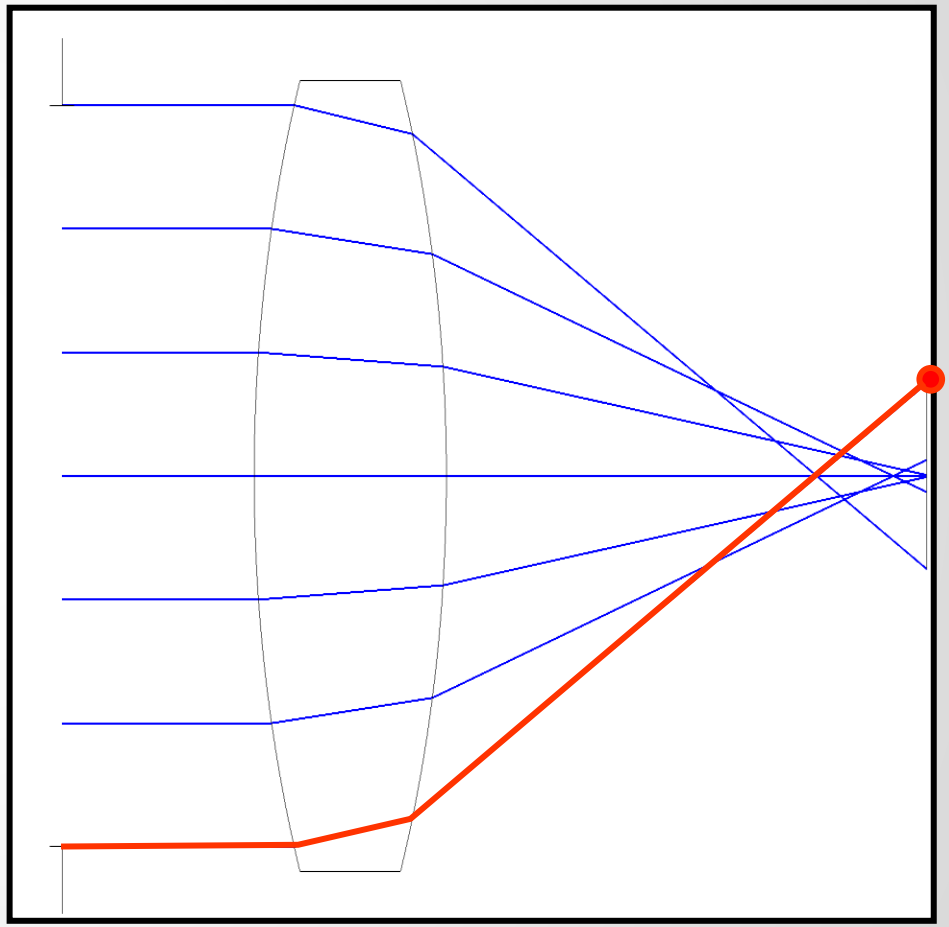
Ray Fan



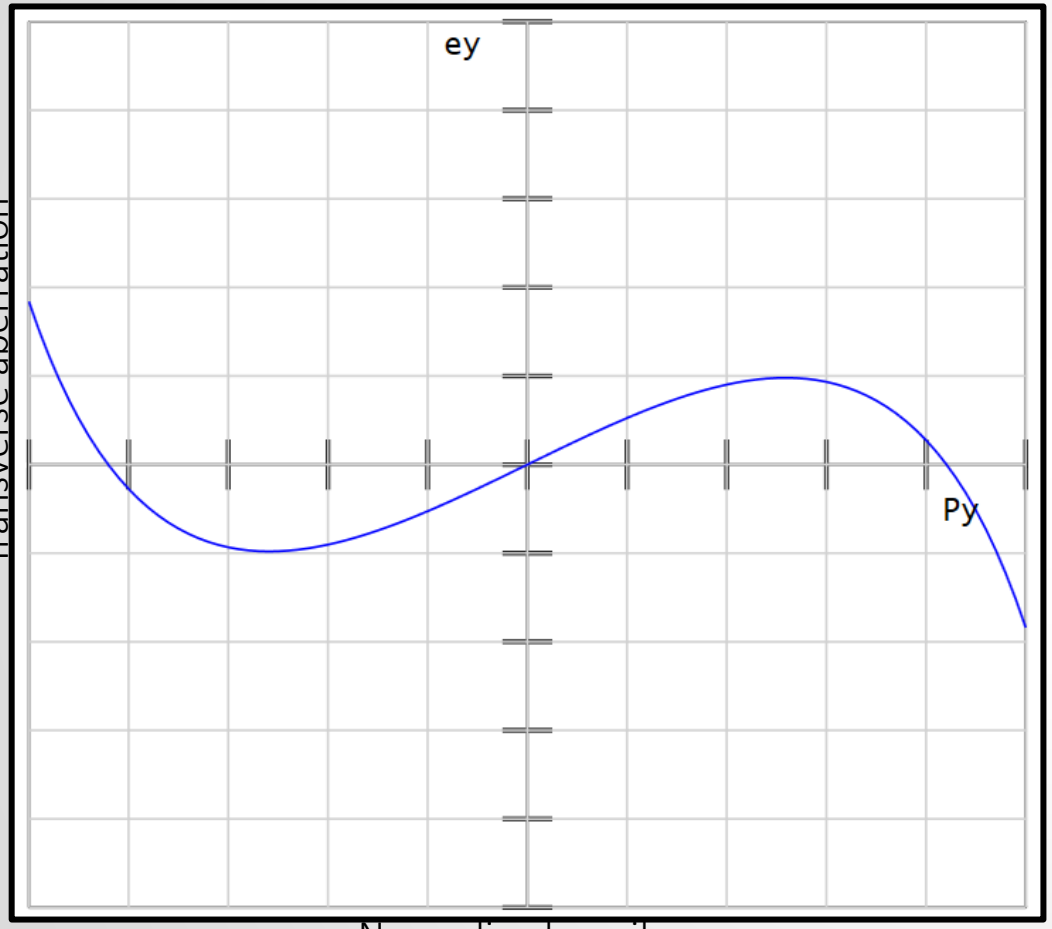


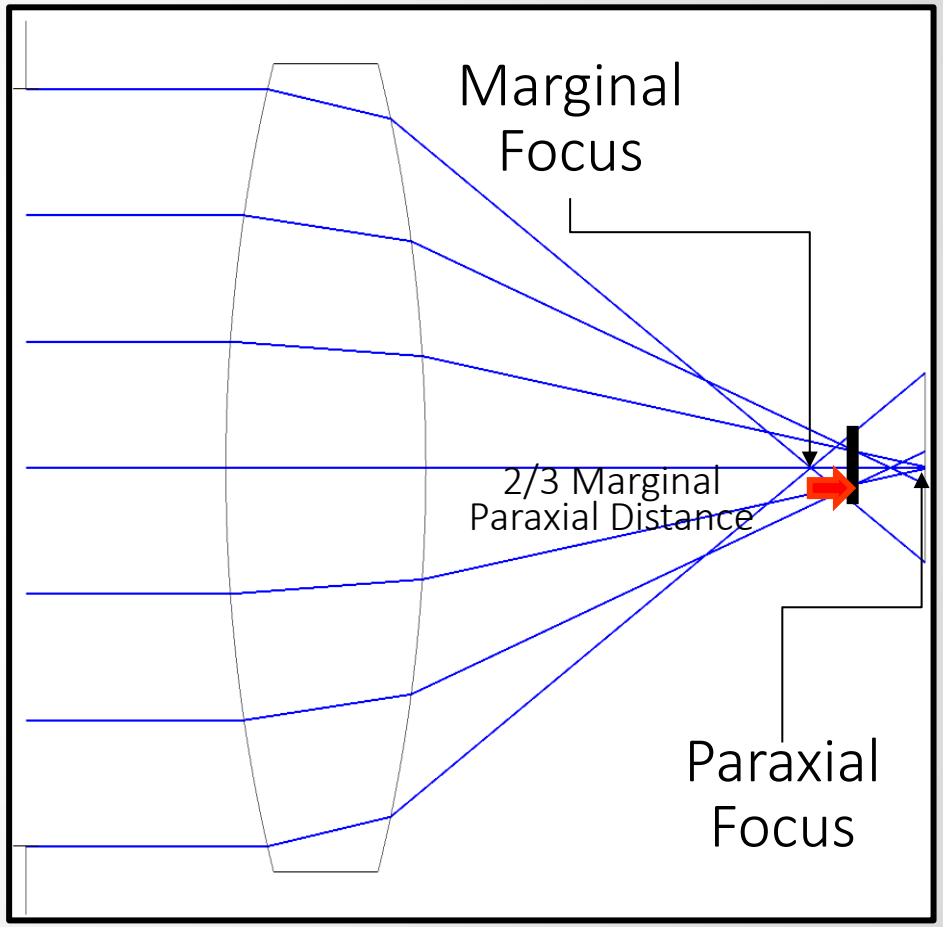
Ray Fan





Minimising Transverse Aberration
Optimum Focus





A small defocus from paraxial focus minimises transverse aberration

Wave Description of Aberration
Wavefront Error (WFE)

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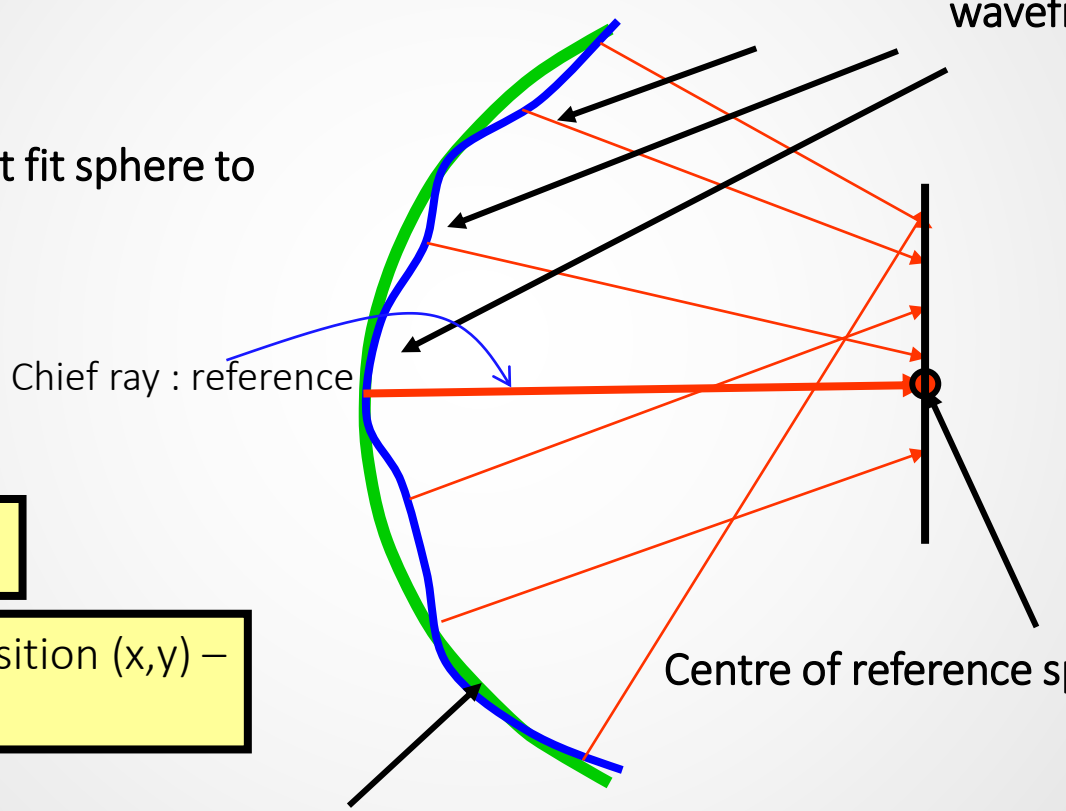
By definition, rays are normal to wavefront

- Reference sphere is best fit sphere to wavefront

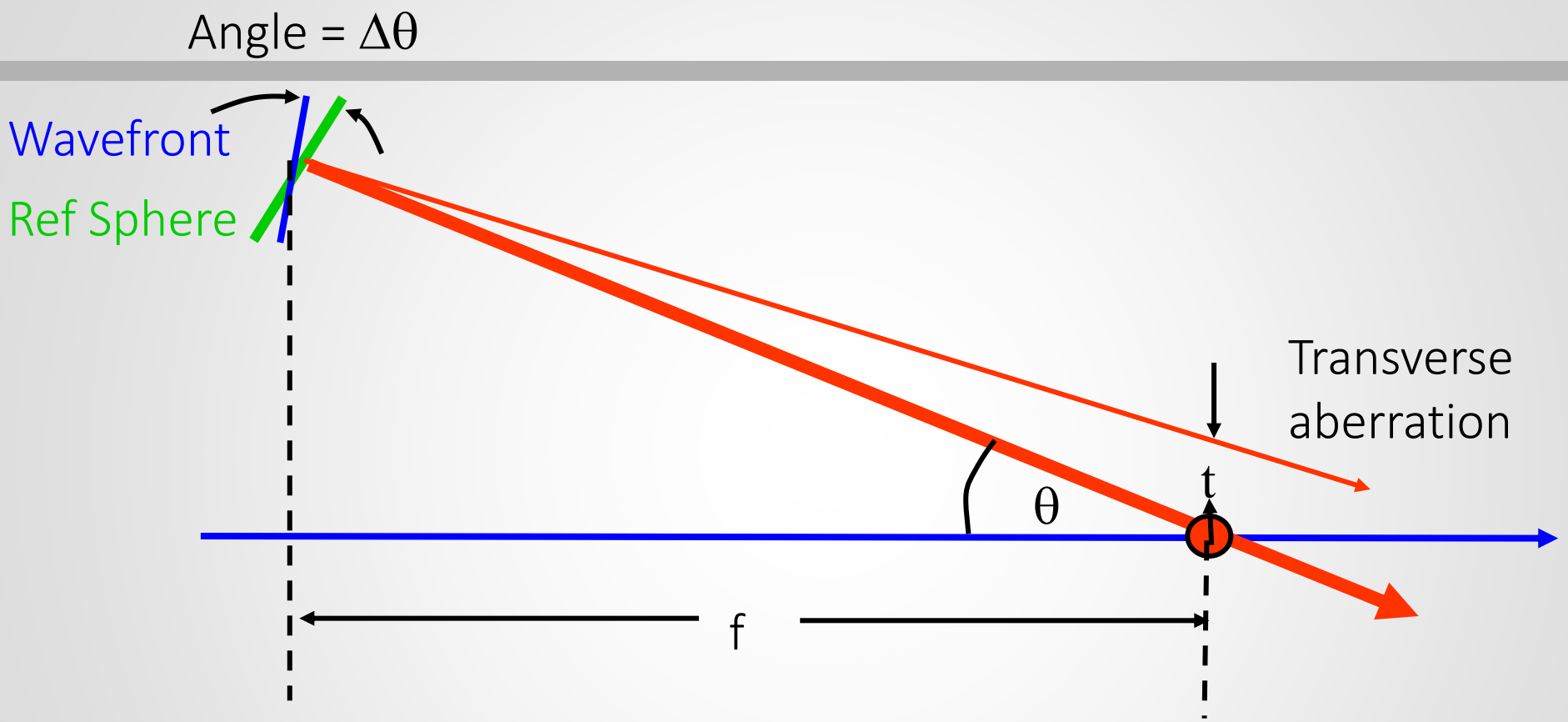


$WFE = n \Delta x$

Δx = ray length at pupil position (x,y) – reference ray length

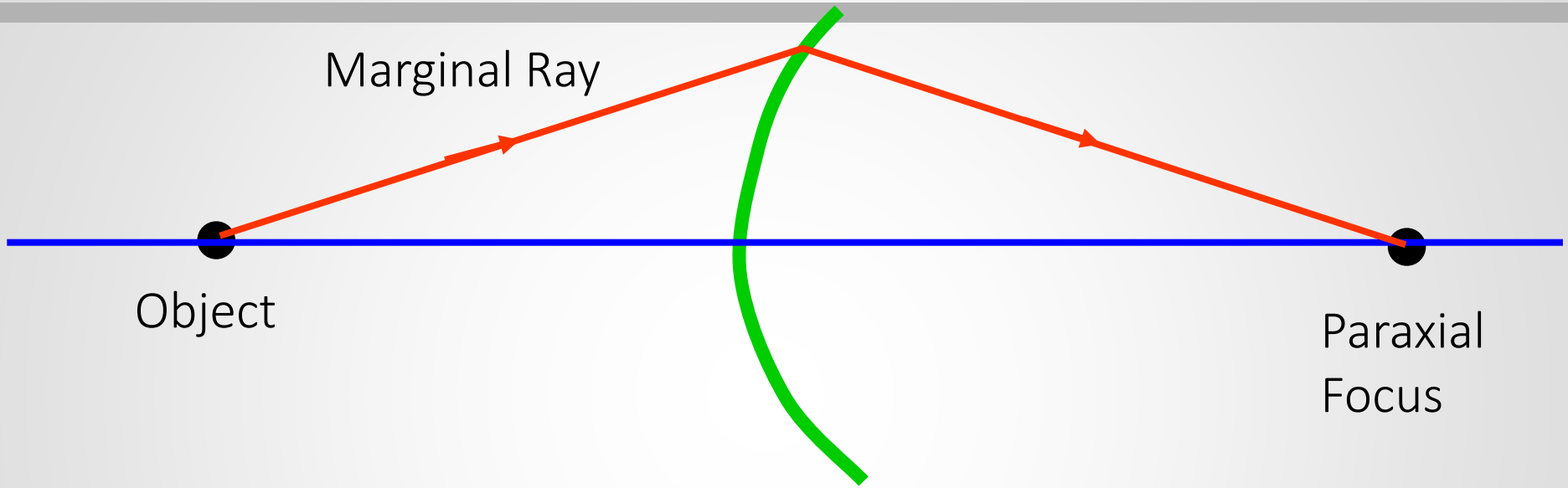
Optical Path Difference between wavefront and reference sphere is WFE

Relation of WFE to TA



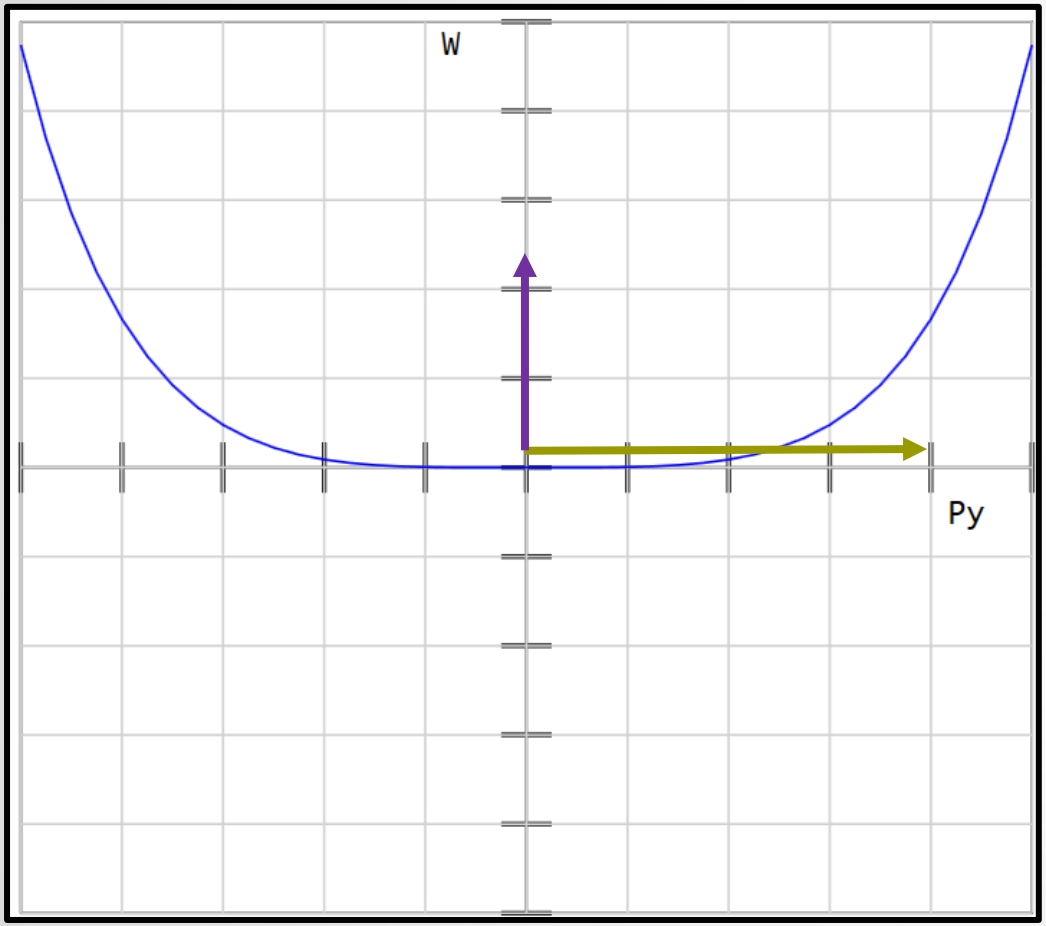
- WFE, is the optical path distance between wavefront and ref sphere taken along normal to ref sphere
- $t = \Delta\theta \,.\, f$

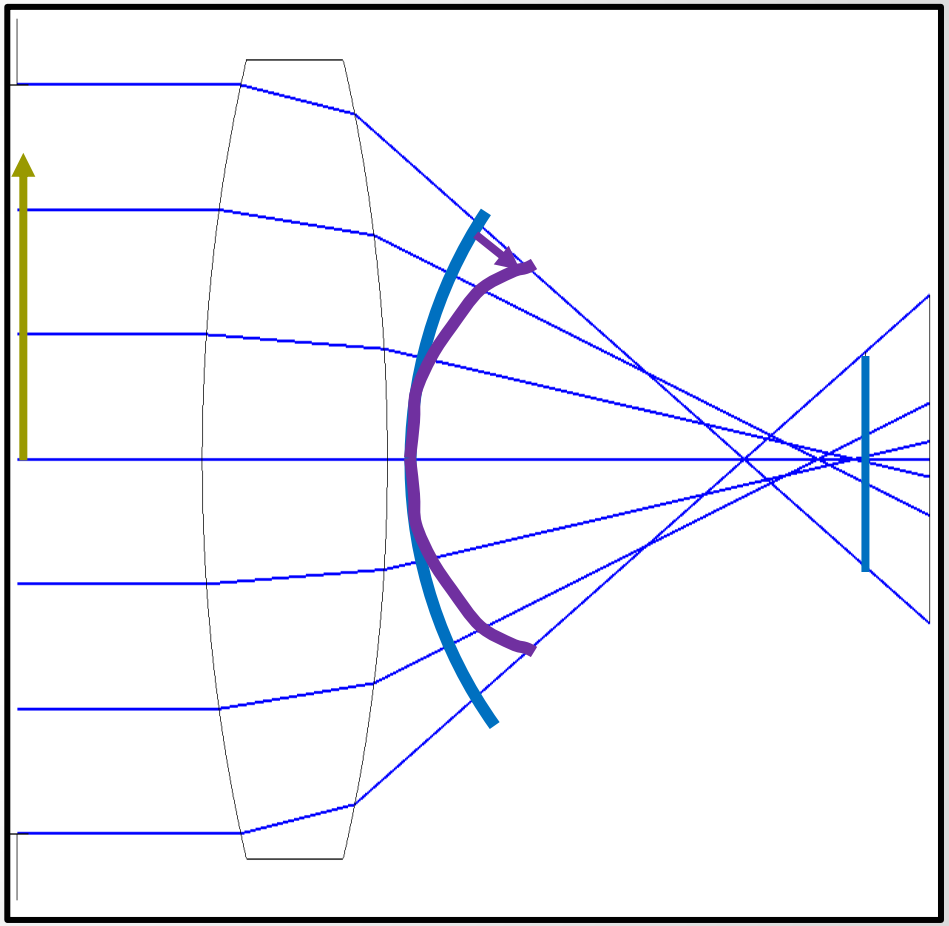
Significance of OPD



- Trace marginal ray to paraxial focus
- Marginal ray does not actually go to paraxial focus
- OPD is the difference in optical path between marginal and chief rays traced in this way.
- OPD is directly related to WFE

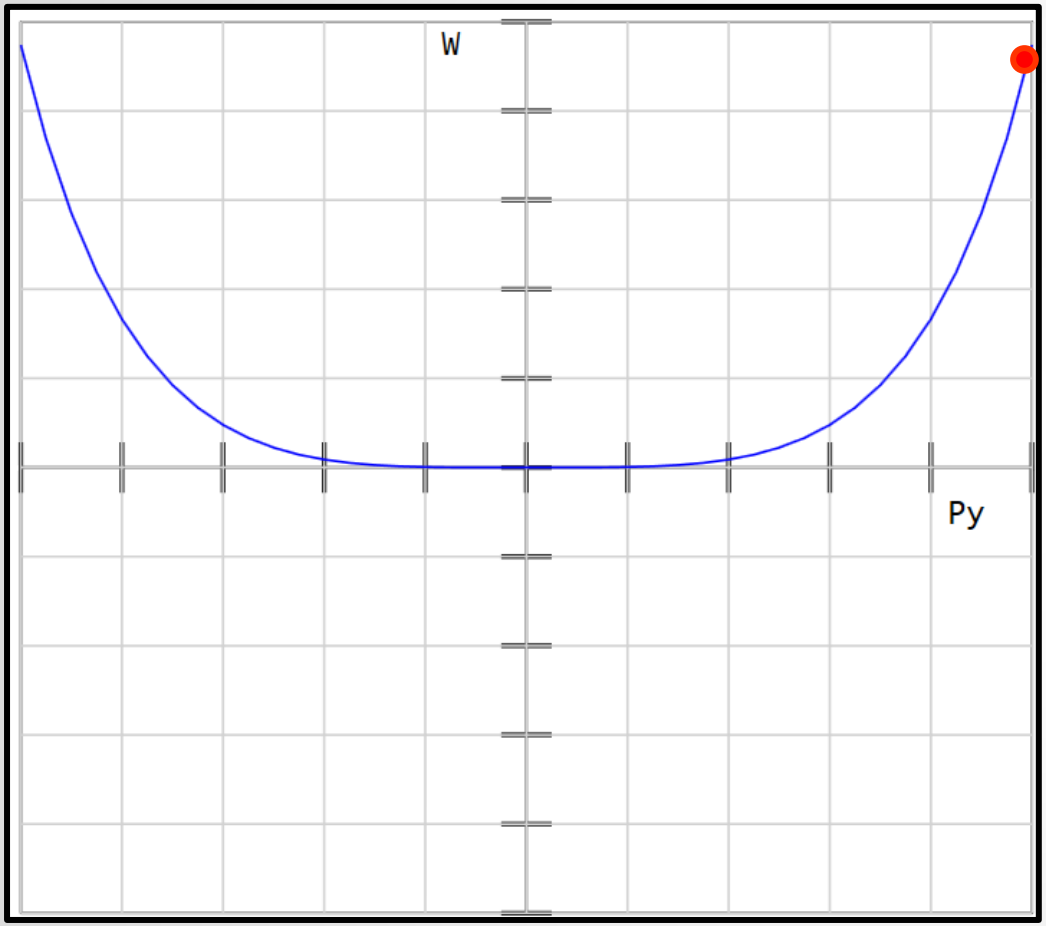
OPD Fan

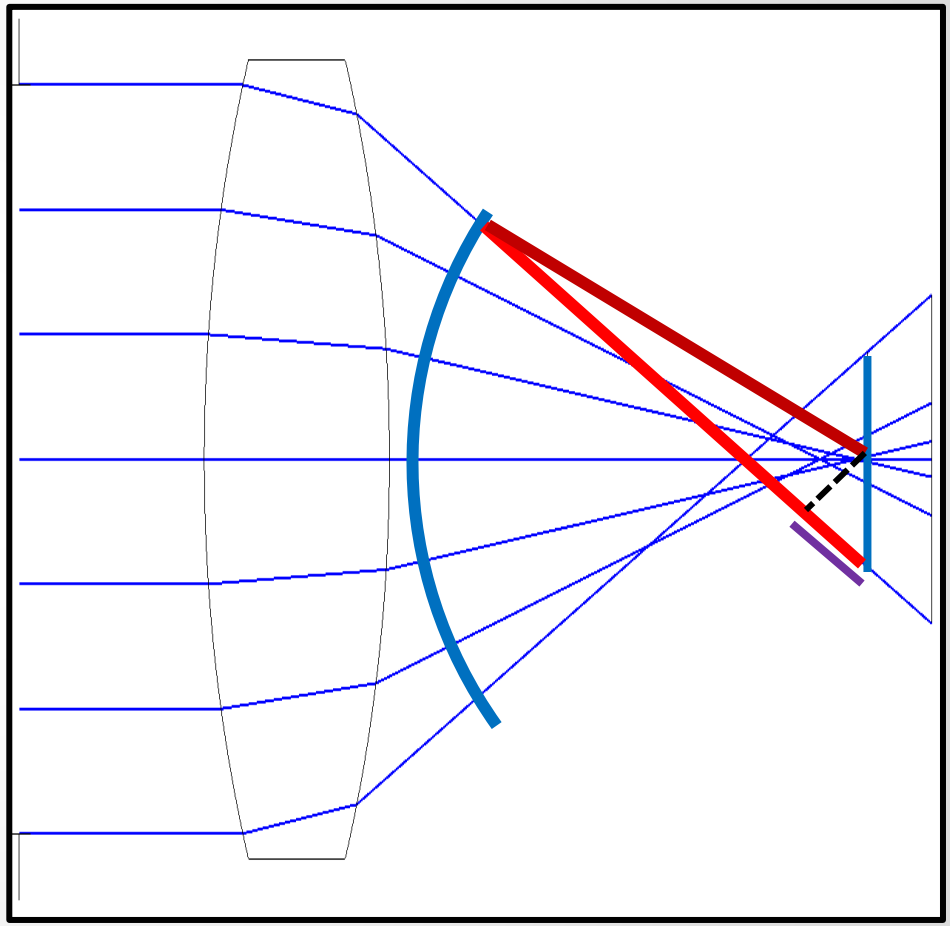




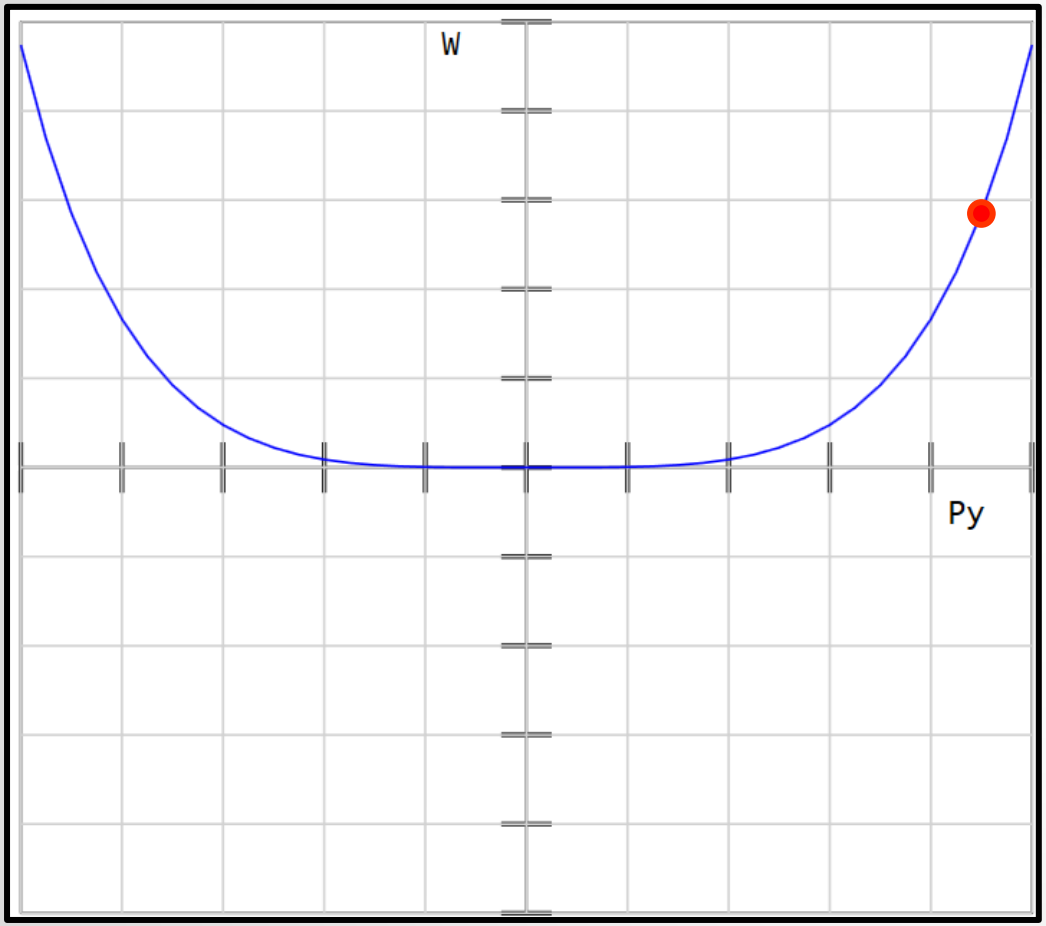
- An OPD (Optical Path Difference) Fan is a plot of OPD vs pupil position

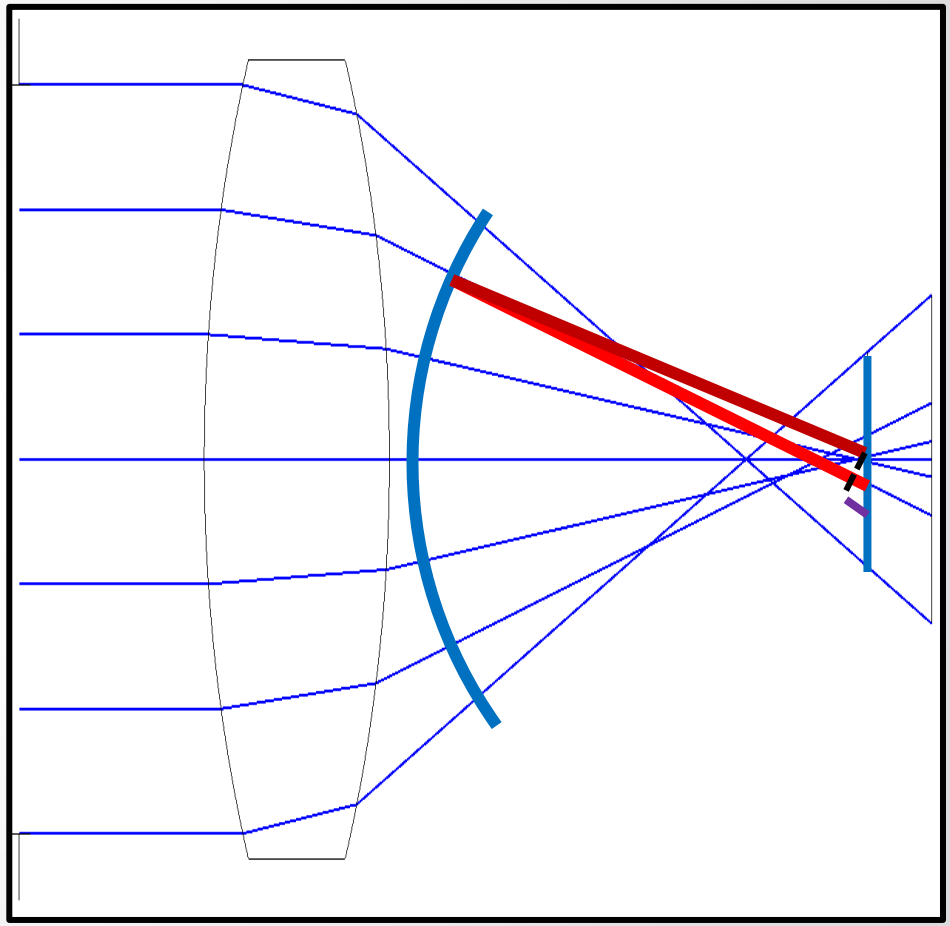
OPD Fan



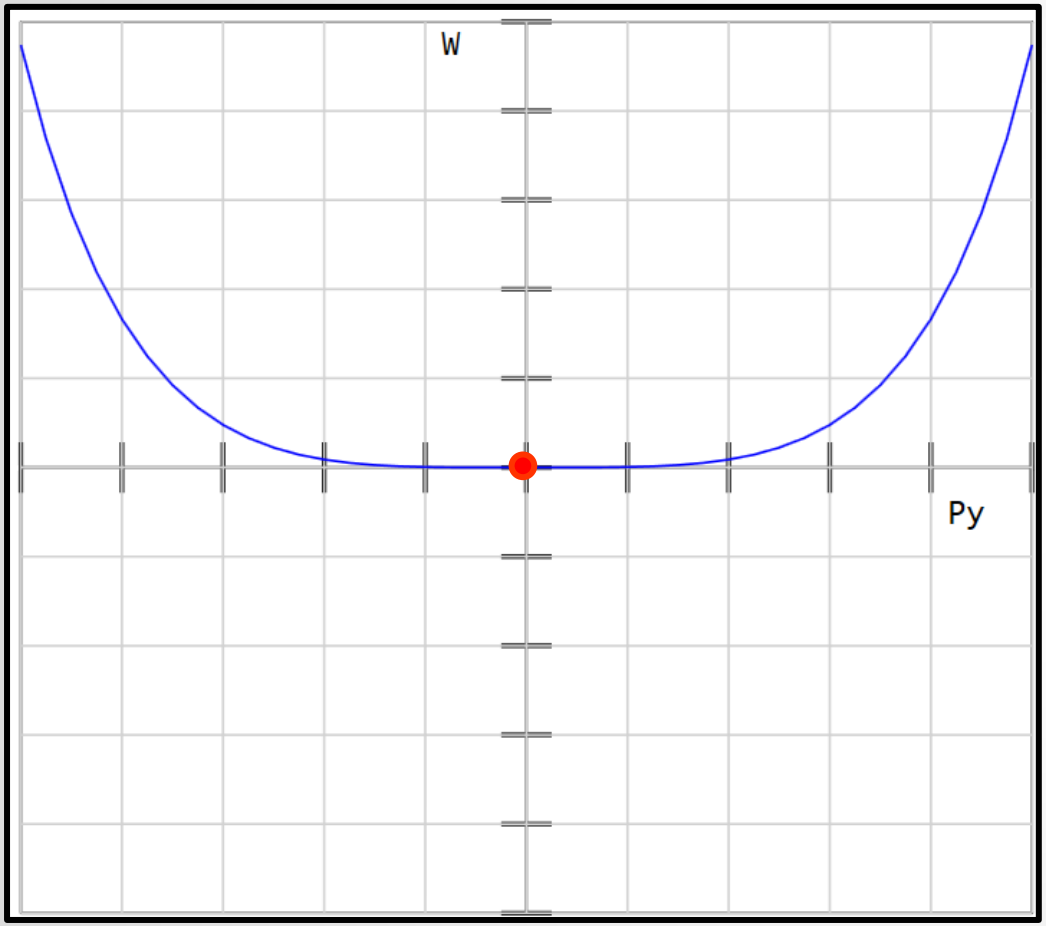


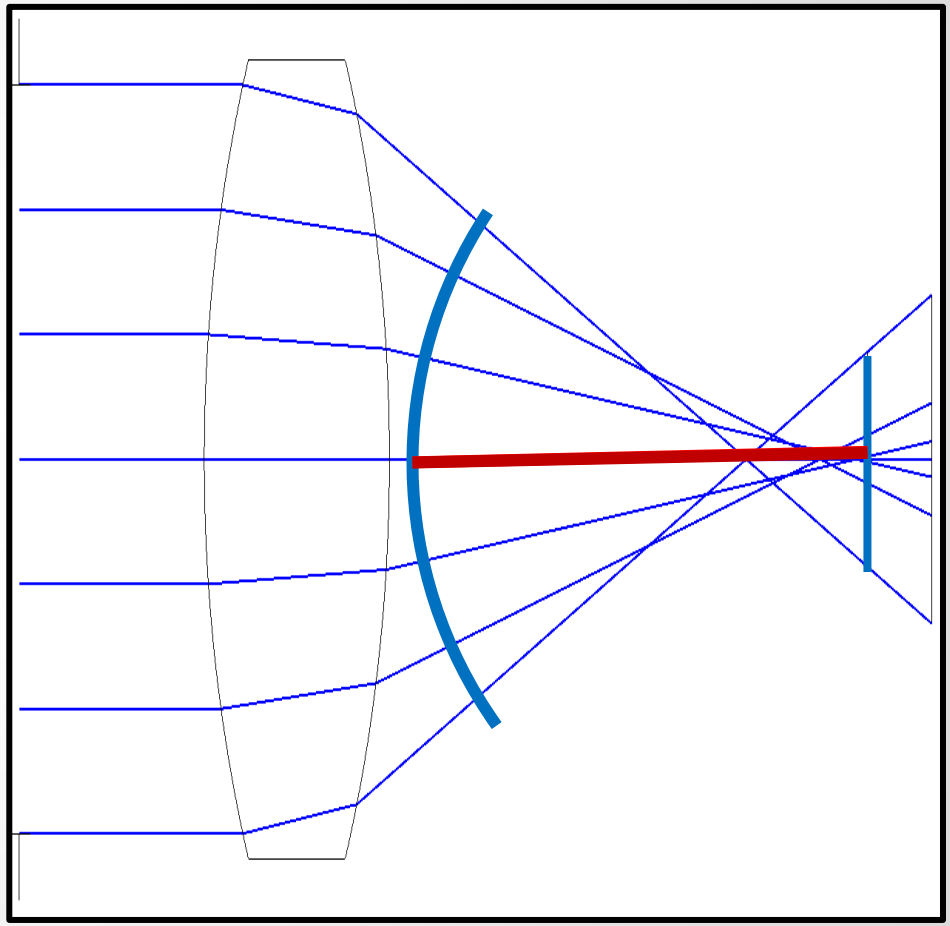
OPD Fan





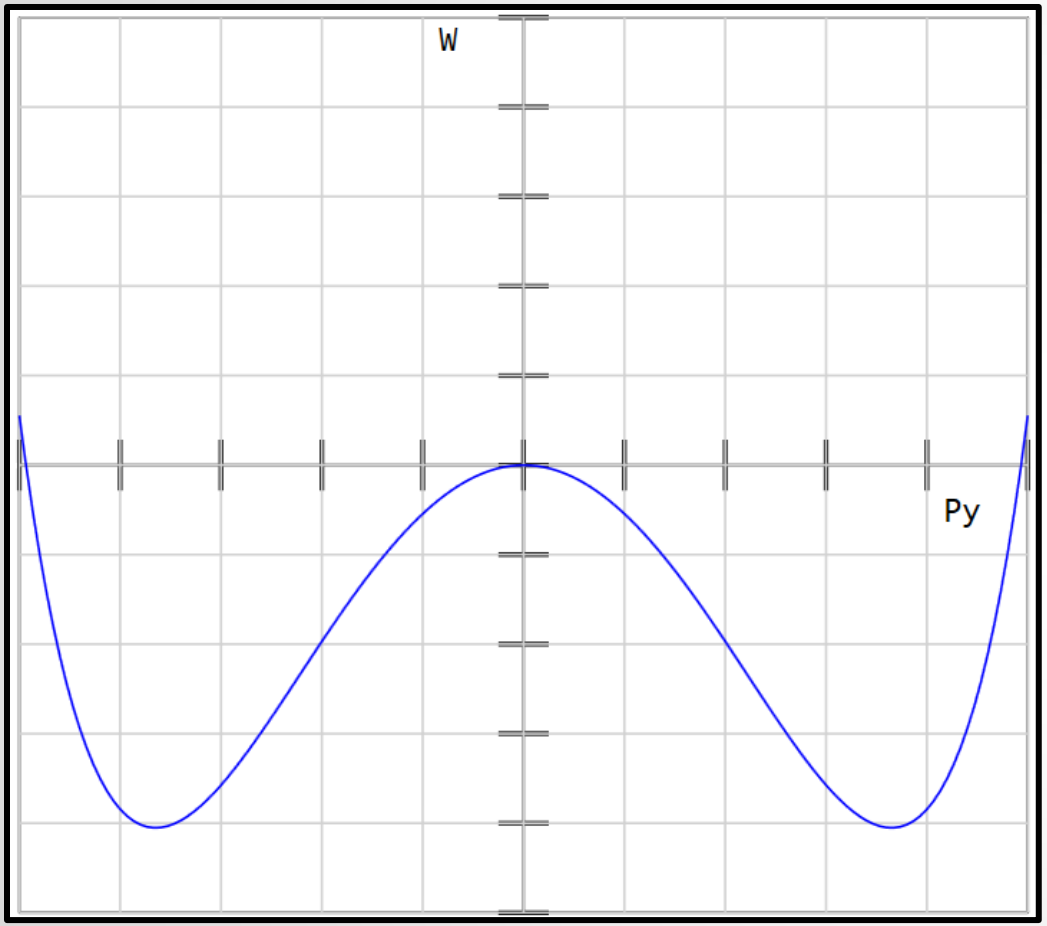
OPD Fan

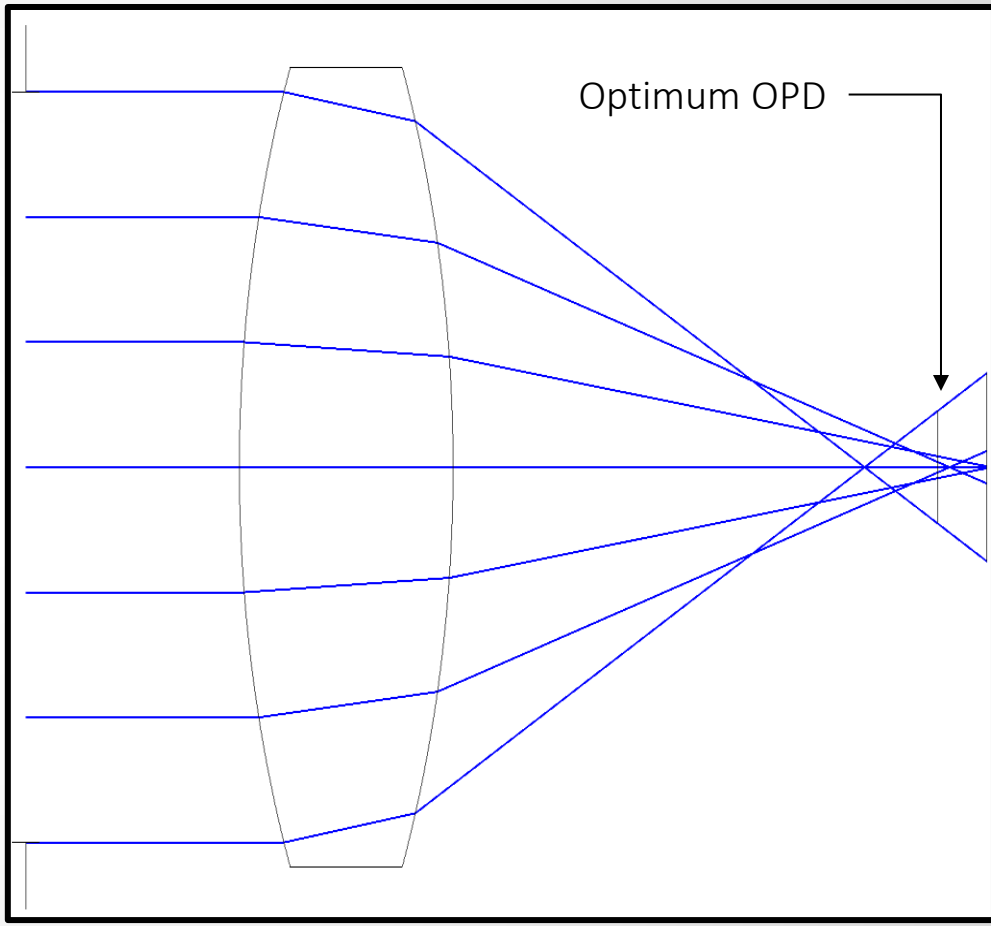




Minimising OPD

Optimum Focus





- the focus position which minimises the OPD is not the same as the one which minimises the spot radius

Geometric and Matrix Optics Summary

Durham University
Centre for Advanced Instrumentation

- Gaussian Optics - Reprise
  - Paraxial approximation
- Matrix Ray Tracing
  - Matrices for simple surfaces
- Matrix ray tracing for complex systems
  - Derivation of cardinal points
  - Spreadsheet analysis
- Breakdown of paraxial approximation
  - Transverse and Longitudinal Aberration
  - OPD and WFE

Ideal Optical System
Paraxial Approximation

