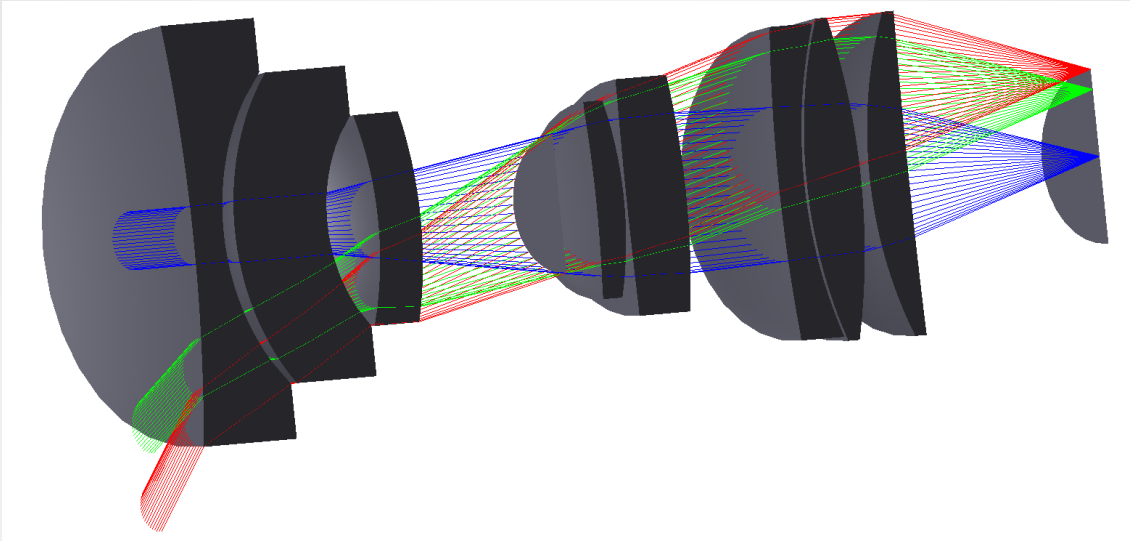

Introduction to Optical Engineering

Michaelmas Term 2026

Lecture 5 – Diffraction and Image Quality

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Centre for Advanced Instrumentation, University of Durham

Optical Engineering - Outline

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- Geometrical Optics
  - General Theory
  - Fermat's Principle
  - Gaussian Optics and paraxial behaviour of Components and Surfaces
- Optical Systems and Aberrations
  - Matrix Ray Tracing
  - Stops and Pupils
  - Introduction to Monochromatic Aberrations
- Monochromatic Aberrations
  - Gauss-Seidel Aberrations
  - Behaviour of Lenses and Mirrors – Aplanatic Points
  - Impact of Pupil on Aberrations
- Aspheric Surfaces and Chromatic Aberration
  - Use of Symmetric Aspheric Surfaces
  - Dispersion and Chromatic Aberration
  - Use of Zernike Polynomials in defining OPD or WFE profiles
- Diffraction and Image Quality
  - Diffraction - 'Near Field' and 'Far Field'
  - Gaussian Beam Propagation
  - Definitions of Image Quality

French joke



Diffraction and Image Quality Summary

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- Theoretical basis for Huygens Principle
  - Rayleigh Formulae
  - Far Field (Fraunhofer) approximation
- Far field diffraction of gaussian beams
- Airy Disc
- Point Spread Function and Strehl Ratio
- Gaussian beam propagation
- Image Quality
  - Geometric Spot
  - PSF
  - MTF

Diffraction

Huygen's Principle

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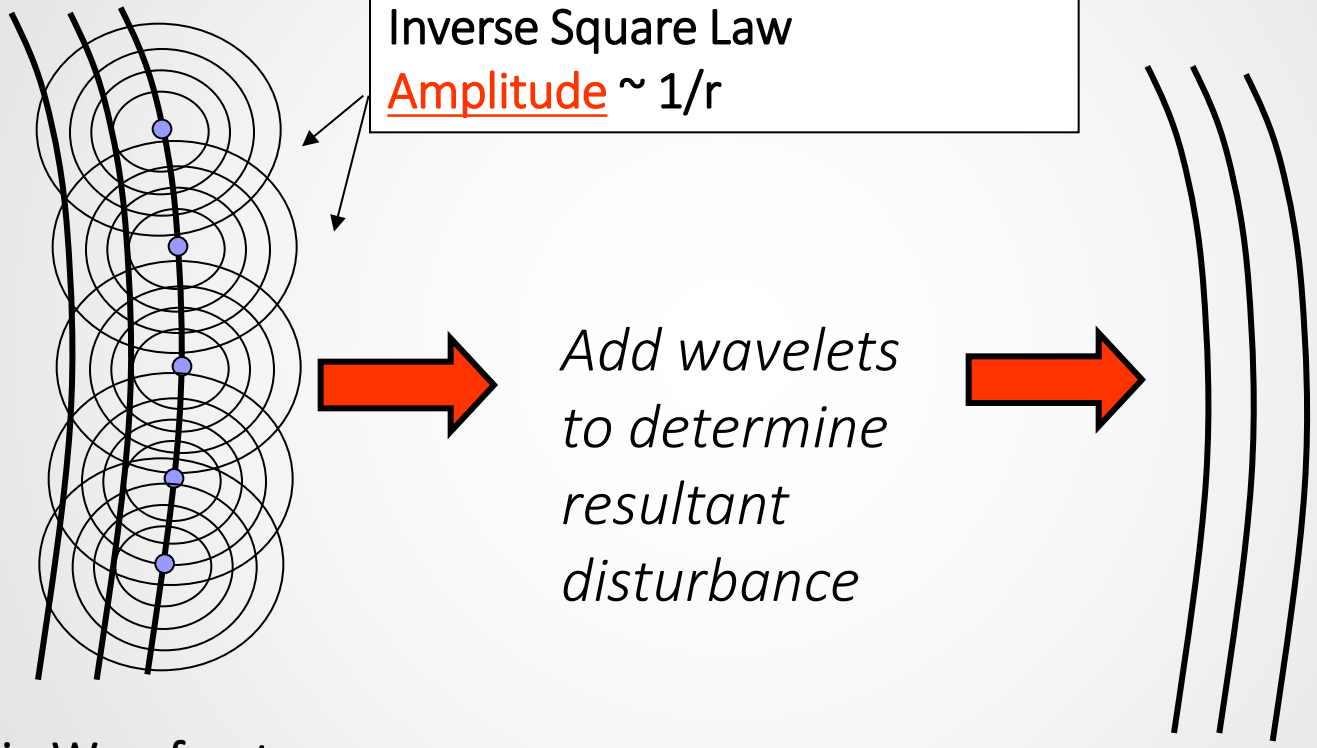
Wavelets – Intensity Follows Inverse Square Law

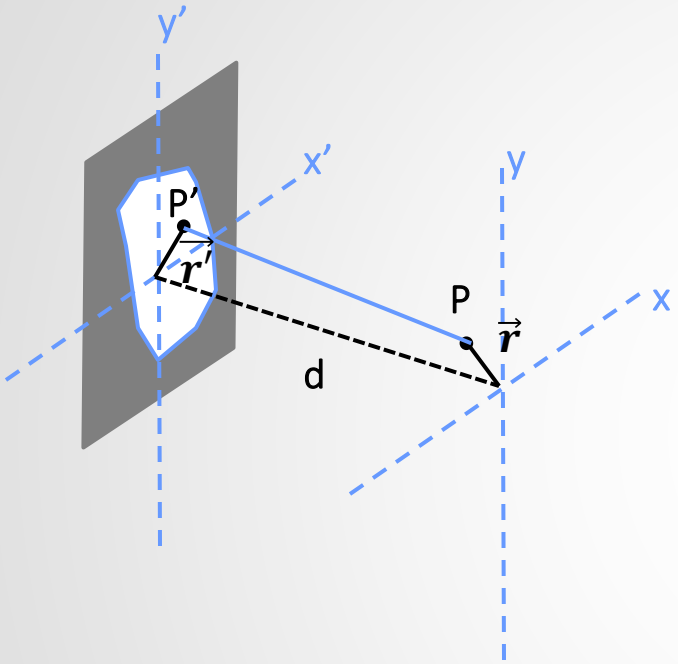
Amplitude ~ 1/r



Add wavelets to determine resultant disturbance

Main Wavefront

Diffraction
Rayleigh-Sommerfeld Formula



Rayleigh-Sommerfeld Formula

$$A(x,y,z) = -\frac{i}{\lambda}\iint A(x',y',z')\frac{e^{ikP'P}}{P'P}\cos\left(\vec{n},\overrightarrow{P'P}\right)dx'dy'$$

Diffraction
Near and Far field approximation

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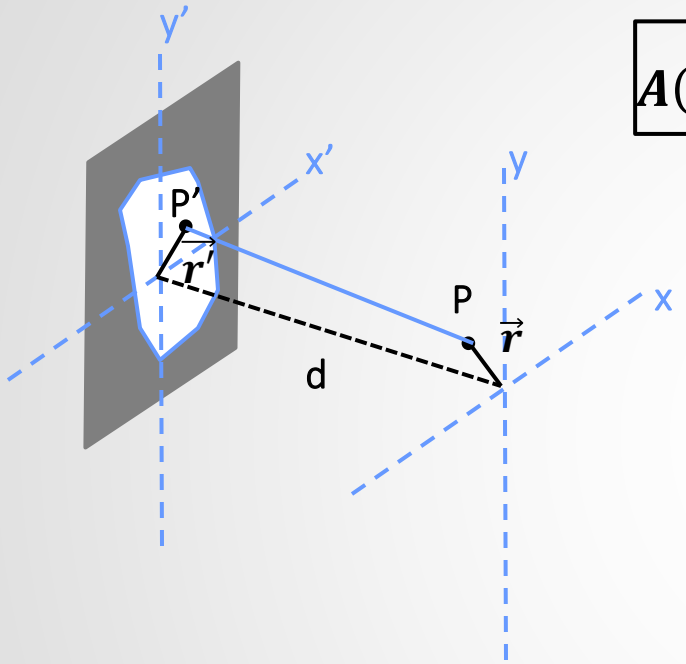


$$A(x,y,z) = -\frac{i}{\lambda}\iint A(x',y',z')\frac{e^{ikP'P}}{P'P}\cos\left(\vec{n},\overrightarrow{P'P}\right)dx'dy'$$

$$P'P = \sqrt{(x-x')^2+(y-y')^2+d^2}$$

$$P'P = \mathrm{d}\sqrt{1+\frac{r^2}{d^2}+\frac{r'^2}{d^2}-2\frac{xx'+yy'}{d^2}}$$

$$P'P = d\left(1+\frac{r^2}{2d^2}+\frac{r'^2}{2d^2}-\frac{xx'+yy'}{d^2}+\epsilon\right)\quad d \gg r, r'$$

$$\cos \approx 1, \sqrt{r^2-r'^2} \ll d,$$

$$A(x,y,d) = -\frac{i}{\lambda d}A_0 e^{ik\frac{r^2}{2d}}\iint A(x',y',0)e^{ik\frac{r'^2}{2d}}e^{-ik\frac{xx'+yy'}{d}}dx'dy'$$

Near Field - Fresnel approximation

$$A(x,y,d) = -i\lambda d A_0 e^{ik\frac{r^2}{2d}}\cdot FT\left[A(x',y',0)e^{ik\frac{r'^2}{2d}}\right]$$

$$e^{ik\frac{r^2}{2d}}, e^{ik\frac{r'^2}{2d}} \approx 1$$

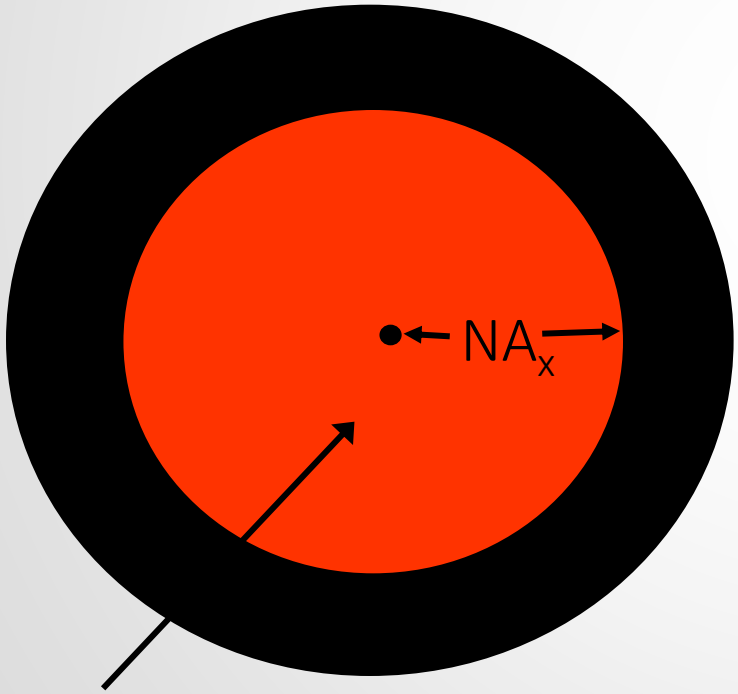
Far Field – Fraunhofer approximation

$$A(x,y,d) \propto FT[A(x',y',0)]$$

Farfield of Uniform Aperture

Airy Disc

- In most instruments, nearfield or farfield defined by uniformly illuminated (circular) aperture
- Find Fourier transform of uniform aperture of NA = NA_x

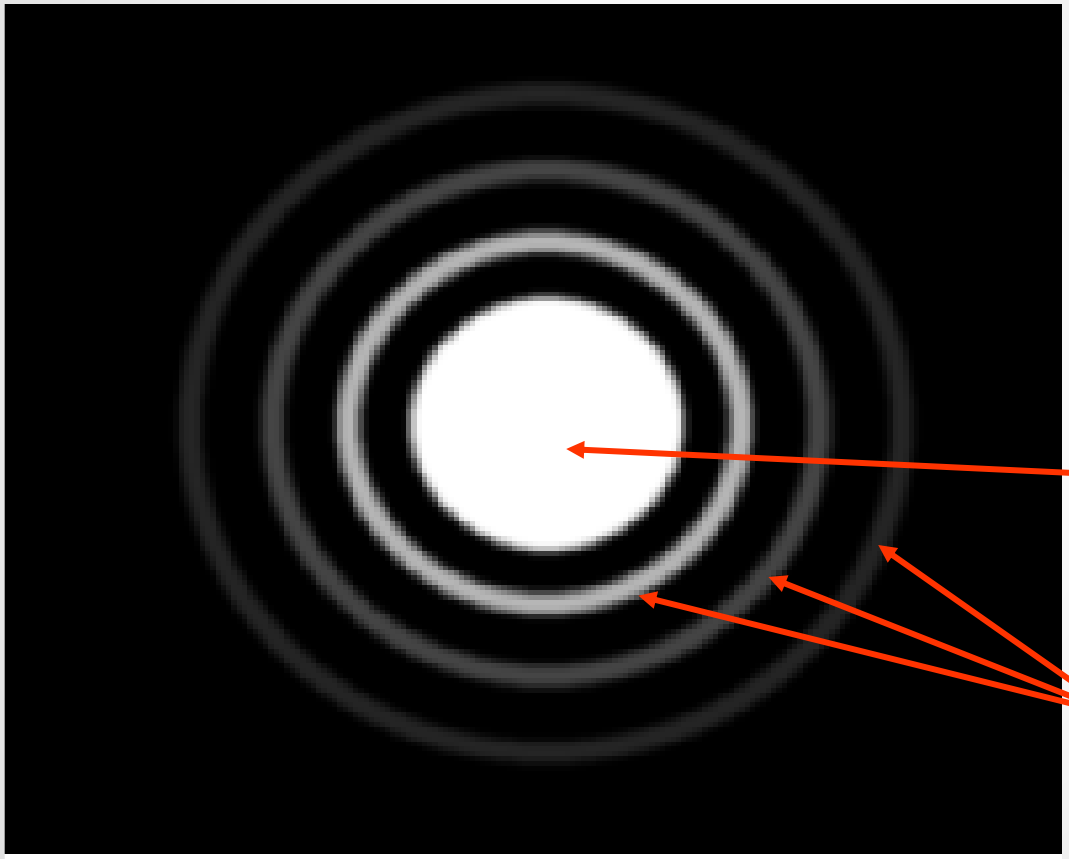


$$I(x) = \left[\frac{2J_1\left(\frac{x}{x_0}\right)}{\left(\frac{x}{x_0}\right)} \right]^2$$

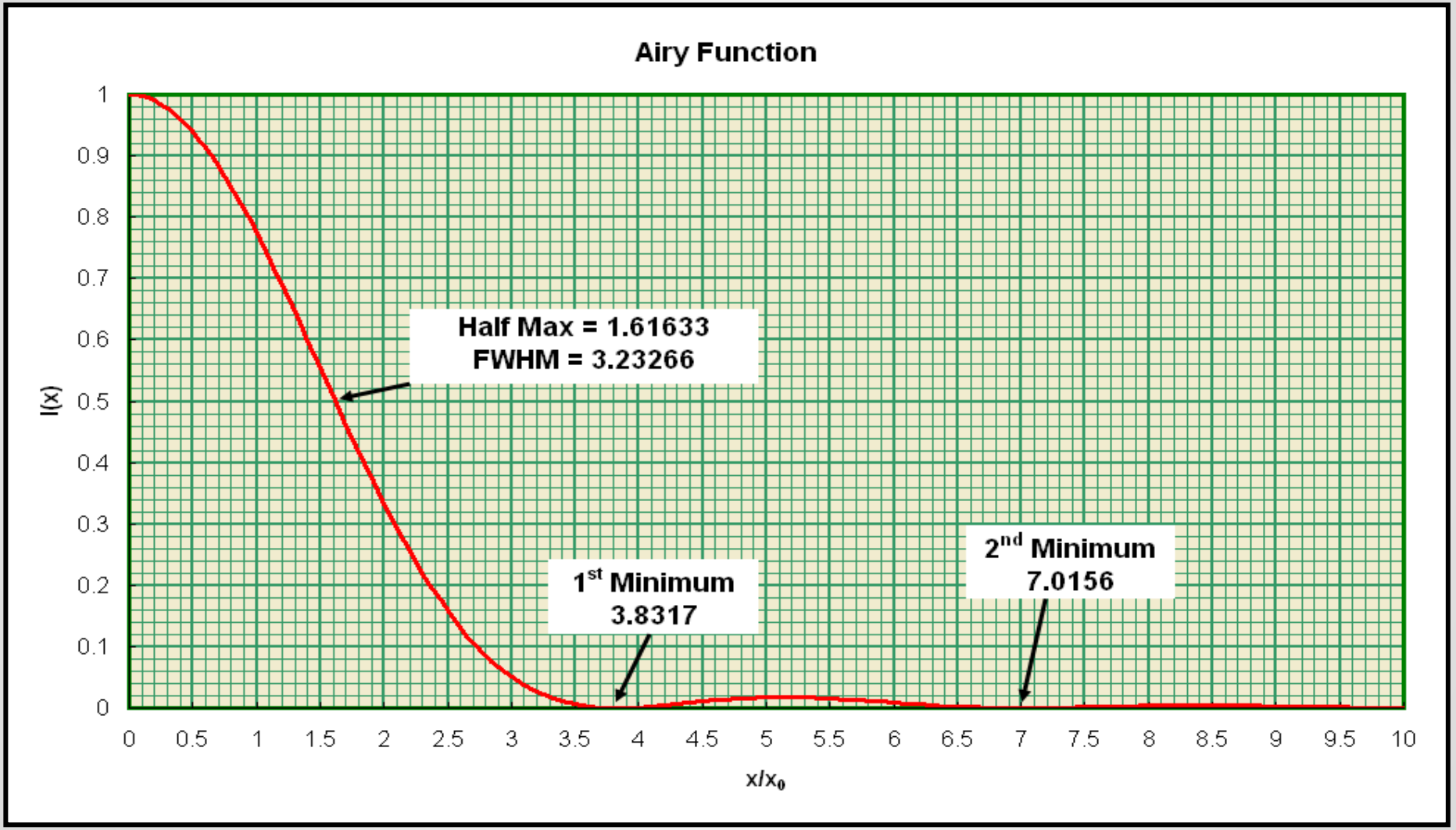
$$x_0 = \frac{\lambda_{vac}}{2\pi NA_x}$$

- Airy Disc
- J_1 = Bessel Function of First Kind

Airy Disc (1) – non Gaussian profile



Airy Disc (2)


Airy Function
I(x)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
x/x0
Half Max = 1.61633
FWHM = 3.23266
1st Minimum
3.8317
2nd Minimum
7.0156

Airy Disc
Size

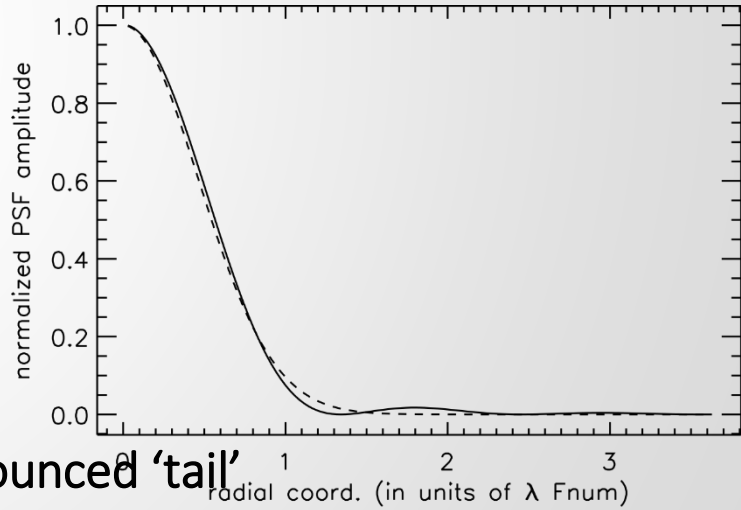
- FWHM is well defined
  - 3.23266 * x_0

$$x_{FWHM} = \frac{1.61633 * \lambda_{vac}}{\pi N A_x} = \mathbf{1.03 \cdot \lambda_{vac} \cdot N}$$

- RMS size is not defined (infinite)
- Optimal fit of Airy to Gaussian gives a Gaussian RMS of:

$$\mathbf{x_{FWHM_Gauss} = 0.84 \cdot \lambda_{vac} \cdot N}$$

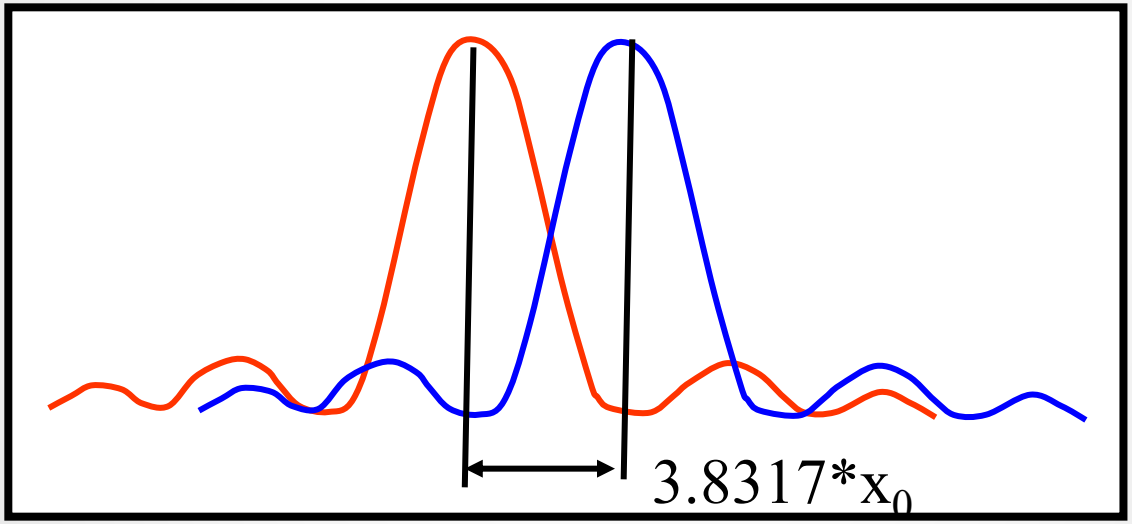
https://en.wikipedia.org/



- Wider than comparable gaussian – Airy has more pronounced 'tail'

Airy Disc
Resolution

- **Rayleigh criterion for (diffraction limited) instrument resolution**
  - Peak of one Airy distribution on **first minimum** of next

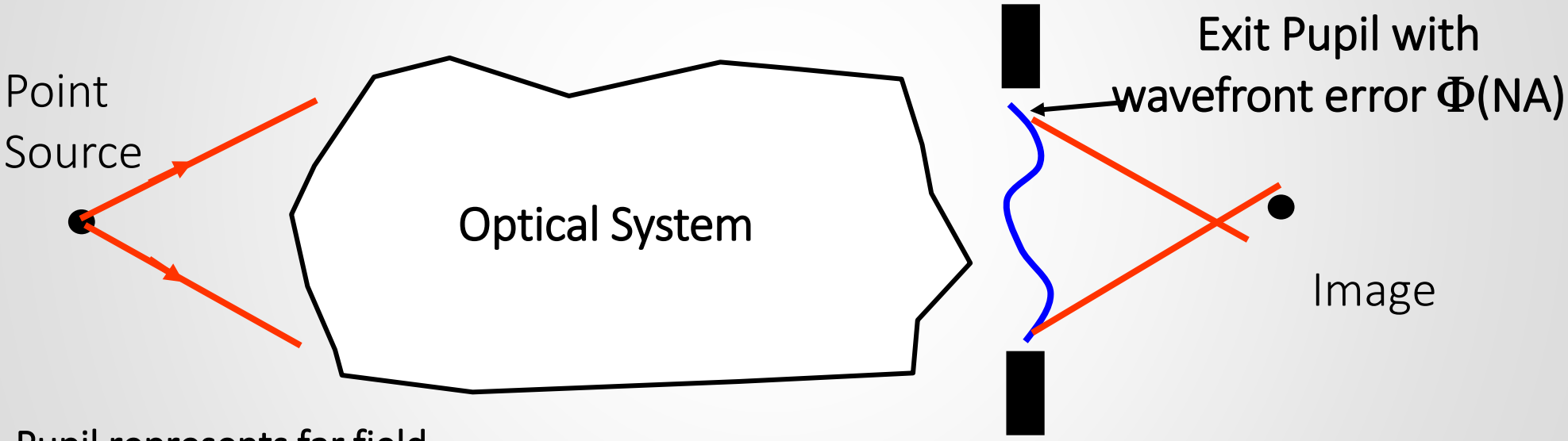


$$x_{resolve} = \frac{1.91585 * \lambda_{vac}}{\pi N A_x} = \frac{0.61 * \lambda_{vac}}{N A_x} = \mathbf{1.22 \cdot \lambda_{vac} \cdot N}$$

Wavefront Map

Wavefront Error Across Circular Pupil

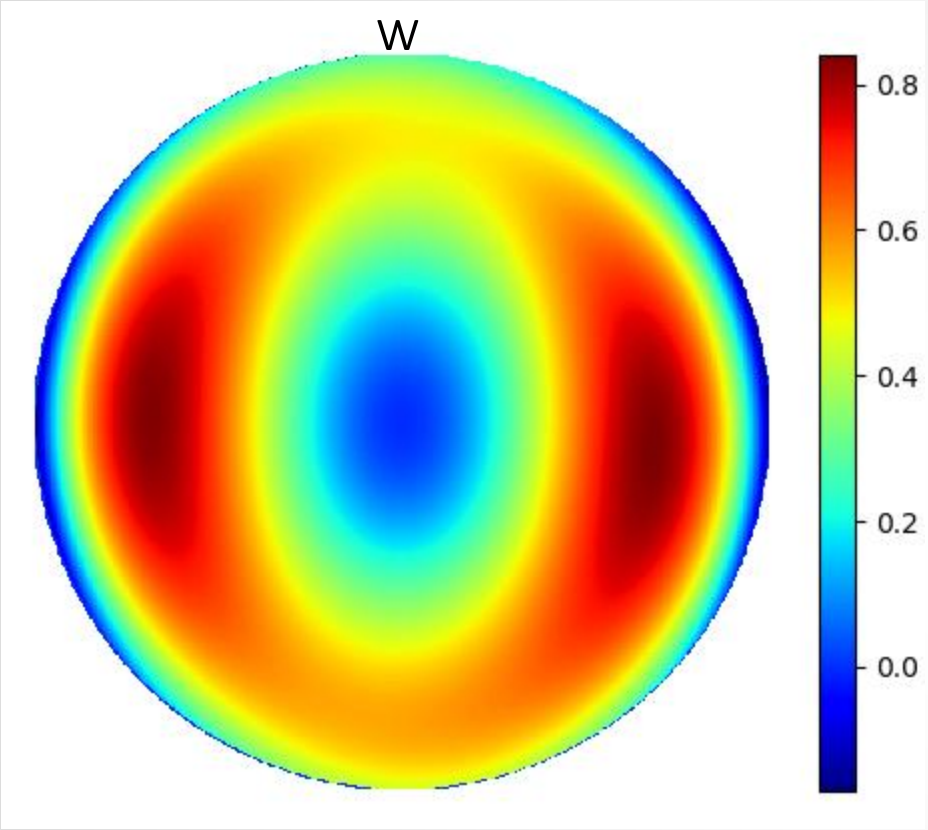
- Project point source from object space using diffraction analysis



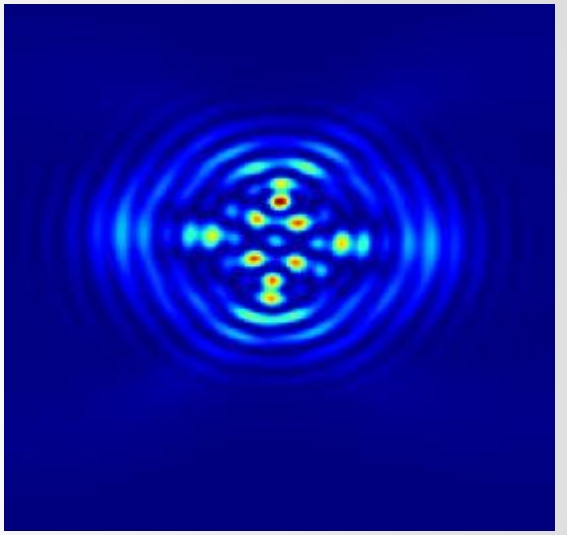
- Pupil represents far field
- In far field approximation, PSF is Fourier transform of pupil function

Point Spread Function

Diffraction Analysis



With padding



Python code :

```
amplitude = pupil_function*numpy.exp(1j*2*numpy.pi*w)
fft = numpy.fft.fft2(amplitude)
psf = numpy.fft.fftshift((fft * fft.conj()).real)
```

pupil_function is :

Point Spread Function
Strehl Ratio

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- Strehl ratio is ratio of peak intensity of aberrated and unaberrated beam and is a measure of the quality of the PSF

$$\boldsymbol{Strehl\ ratio} = \left(\frac{\boldsymbol{FT}\left[e^{ik\cdot\Phi}\right]}{\boldsymbol{FT}\left[e^{ik\cdot\Phi_{*0}}\right]} \right)^2$$

- If $\Phi \ll \lambda$, then the system is close to be diffraction limited $\quad \boldsymbol{\Phi_{RMS}} < \frac{\boldsymbol{\lambda}}{\boldsymbol{14}} \boldsymbol{RMS}$

$$\boldsymbol{variance} = \boldsymbol{RMS^2} = \langle \boldsymbol{\Phi^2} \rangle - \langle \boldsymbol{\Phi} \rangle^2$$

$$\boldsymbol{Strehl} \approx 1 - \left(\frac{2\pi}{\lambda}\right)^2 \cdot \boldsymbol{variance}$$

Strehl > 0.8 => Diffraction Limited

Strehl < 0.8 => Not Diffraction Limited

Strehl Ratio

Representation of WFE by Zernikes

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- Total RMS WFE is simply the RSS sum of the Zernike Polynomial Coefficients that describe the WFE

$$\mathbf{W}(\boldsymbol{\rho},\boldsymbol{\theta}) = \sum_{n,m} \boldsymbol{a}_{n,m} \boldsymbol{Z}_n^m(\boldsymbol{\rho},\boldsymbol{\theta})$$

$$\mathbf{variance} = \sum_{n,m} \left(\boldsymbol{a}_{n,m}\right)^2$$

$$\boldsymbol{Strehl} \approx 1 - \left(\frac{2\pi}{\lambda}\right)^2 \cdot \sum_{n,m} \left(\boldsymbol{a}_{n,m}\right)^2$$

Waves & Beam Propagation

Fundamental Equations

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$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}=\frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Maxwell's Equation – derived from fundamental laws of electromagnetics.
(E is the electric field and c is the speed of light, n the refractive index)

$$c=\sqrt{1/\mu_0\varepsilon_0}$$

$$n=\sqrt{\mu\varepsilon}$$

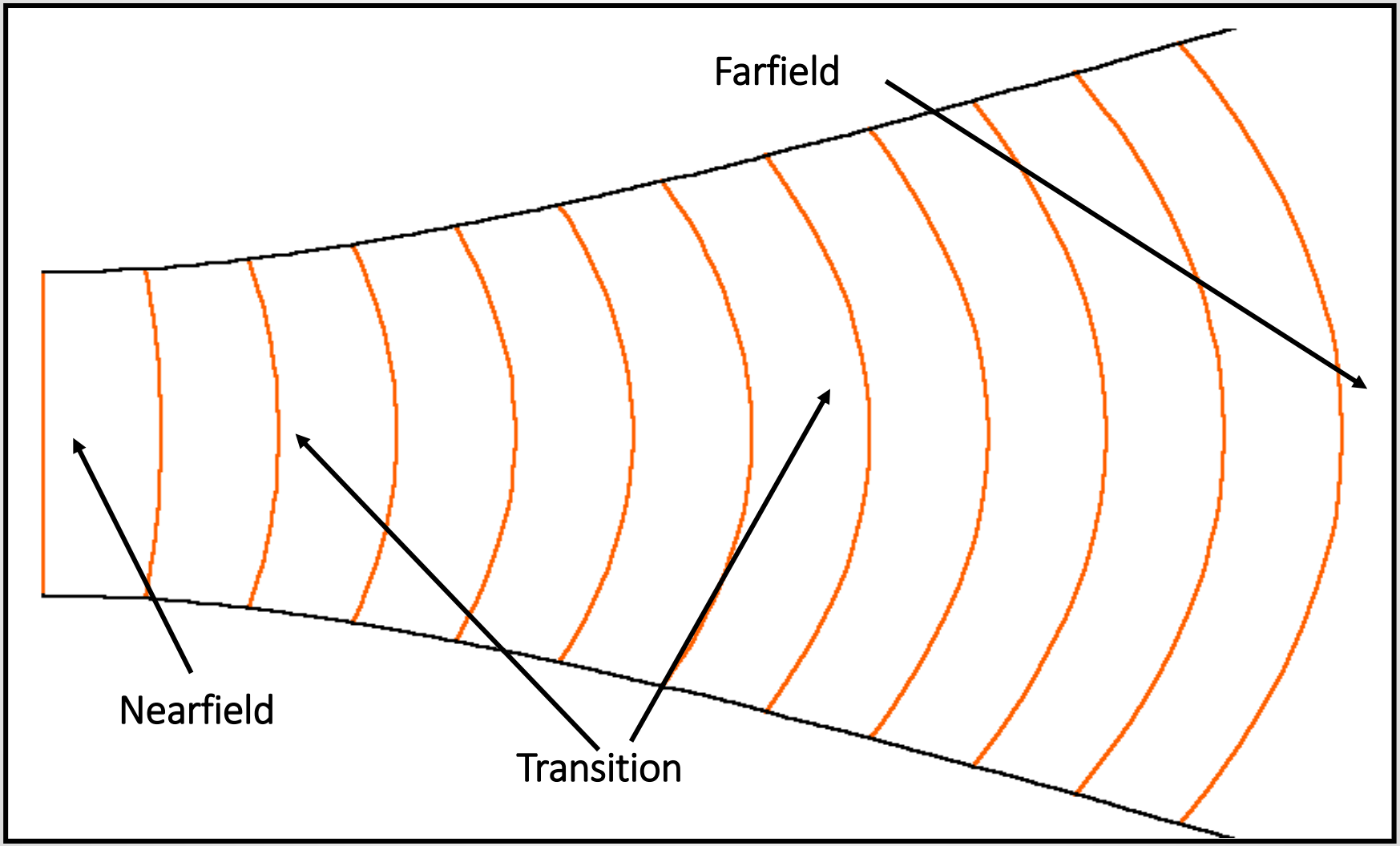
$$E(x,y,z)=A(x,y)e^{j(\omega t-kz)}$$

Paraxial Helmholtz Equation:

$$\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}-j2k\frac{\partial A}{\partial z}=0$$

- Describes beam propagation in paraxial approximation.
- A is the so called envelope and varies slowly in comparison to the wavelength
- This is the slowly varying envelope approximation

Gaussian Beam Propagation



Gaussian Beam Description

- In amplitude, expressed as an envelope

$$E(x,y,z) = E_0 e^{-\left[\frac{x}{x_0}\right]^2} e^{j(\omega t - kz)}$$

- Near field <u>intensity</u> distribution often given as Gaussian

$$I(x) = I_0 e^{-2\left[\frac{x}{x_0}\right]^2}$$

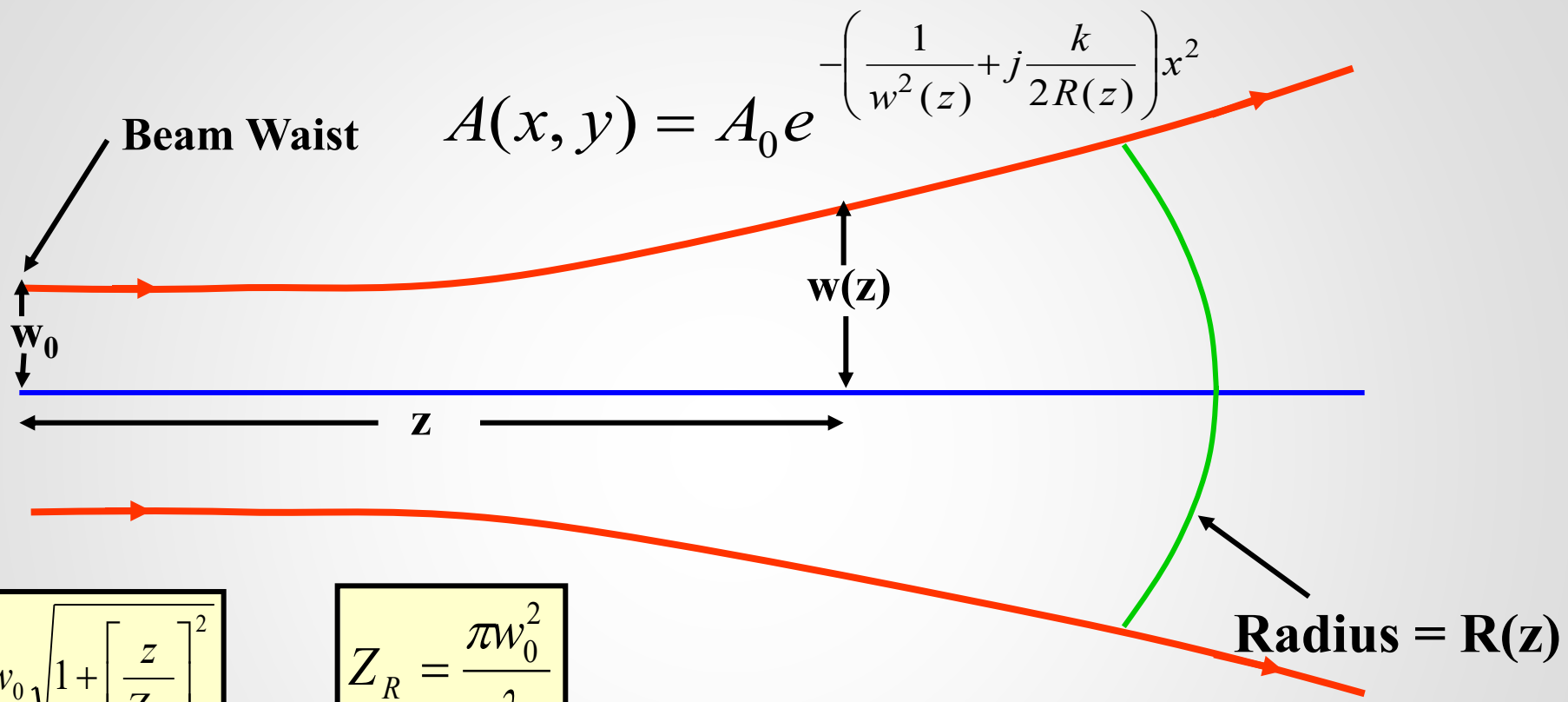
- However, x_0 varies slowly with z *(written as: w(z))* – envelope is:

$$A(x,y) = A_0 e^{-\left(\frac{1}{w^2(z)} + j\frac{k}{2R(z)}\right)x^2}$$

R(z) is the wavefront radius

- More details in Born & Wolf, Principles of Optics.

Gaussian Beam Description



$$w(z) = w_0\sqrt{1+\left[\frac{z}{Z_R}\right]^2}$$

$$Z_R = \frac{\pi w_0^2}{\lambda}$$

$$R(z) = z + \frac{Z_R^2}{z}$$

$$NA_0 = \frac{\lambda}{\pi w_0}$$

Z_R is called the Rayleigh Distance

λ is wavelength in medium i.e. λ_0/n

Rayleigh Distance

- Rayleigh distance provides a useful estimate in distinguishing between near and far fields
- $Z << Z_R$ - Nearfield
- $Z >> Z_R$ – Farfield
- $Z \sim Z_R$ – 'Transition'
- Rayleigh distance provides estimate of depth of focus for diffraction limited system

Manipulation of Gaussian Beams

Impact of Optical System Propagation

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- Complex parameter, q, provides complete description of gaussian beam
- $q(z) = z + j\,Z_r$ $\qquad \frac{1}{q(z)} = \frac{1}{R(z)} - \mathrm{j}\frac{\lambda}{\pi w^2(z)}$
- Paraxial optical system represented by matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- Uniquely determine new gaussian beam from following relation that enables calculation of new complex q value:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

Physical Optics Propagation

Gaussian-Hermite Polynomials

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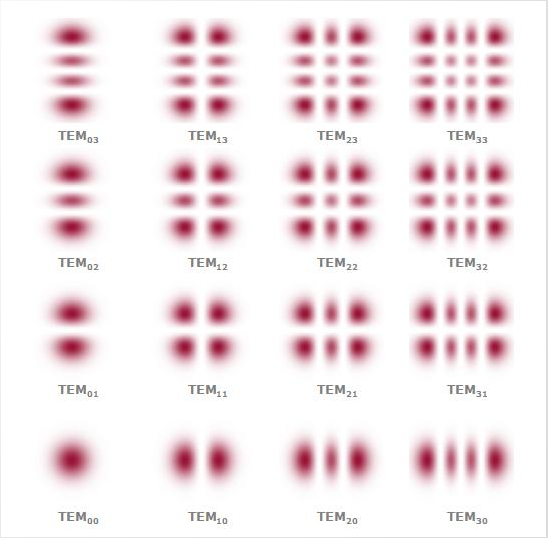
- Gaussian beam not the only solution to the Paraxial Helmholtz Equation
- Gaussian-Hermite Polynomials represent complete solution set for the Paraxial Equation => electric field amplitude distributions does not change during propagation
- Any solution can be represented by a superposition of terms
- Useful for beam propagation modelling



https://www.rp-photonics.com/hermite_gaussian_modes.html

Image Quality
Some Measures

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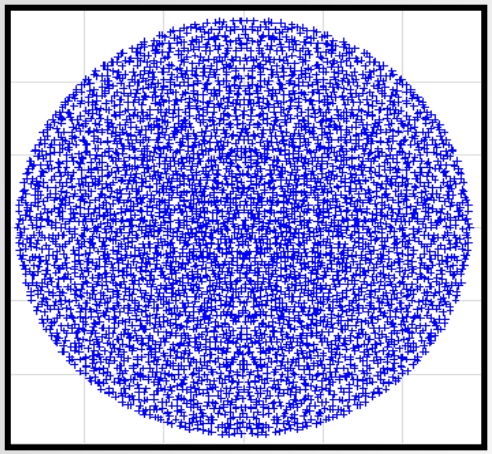
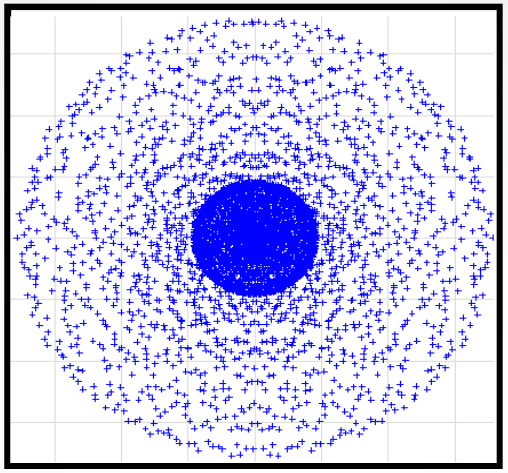
- **RMS spot size and distribution**
  - Related to geometrical optics
  - Distribution gives some indication of underlying aberrations
  - Not so useful where system is (nearly) diffraction limited
- **Point Spread Function (PSF)**
  - Related to physical optics
  - Gives the real image intensity distribution in the presence of diffraction
  - Peak intensity of the PSF gives the so called 'Strehl Ratio'
- **Strehl Ratio**
  - A measure of how close the system is to diffraction limited
  - Strehl ratio of unity is perfect
  - Strehl ratio of anything greater than 0.85 is considered diffraction limited
- **Modulation Transfer Function (MTF)**
  - Measures the contrast ratio of an image of a sinusoidally varying pattern as a function of spatial frequency

Image Quality

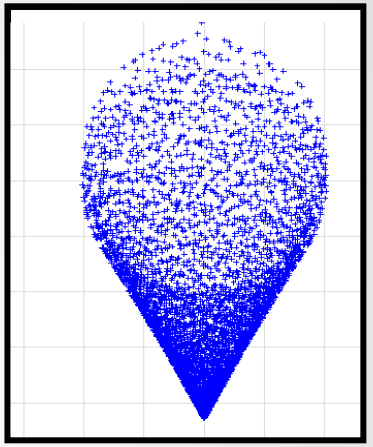
Spot Size



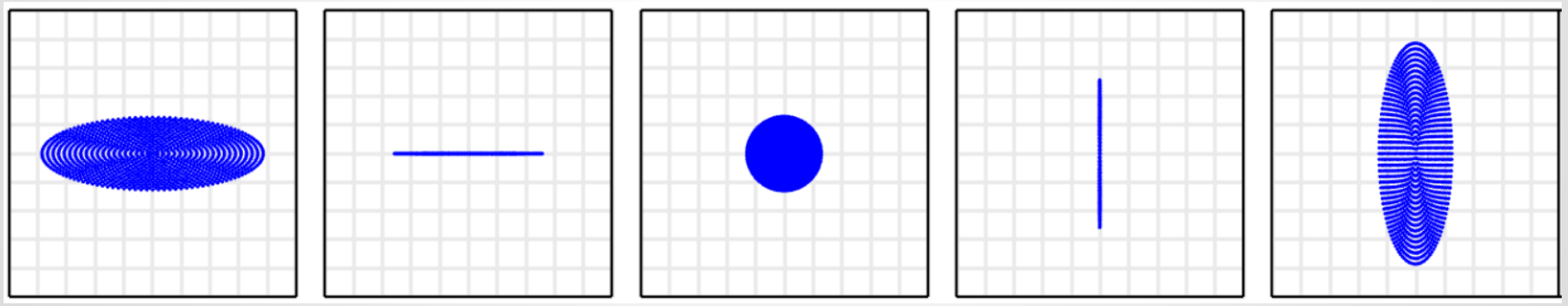
Defocus



Spherical Aberration

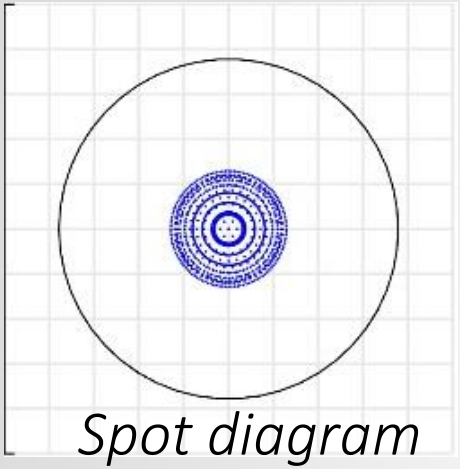


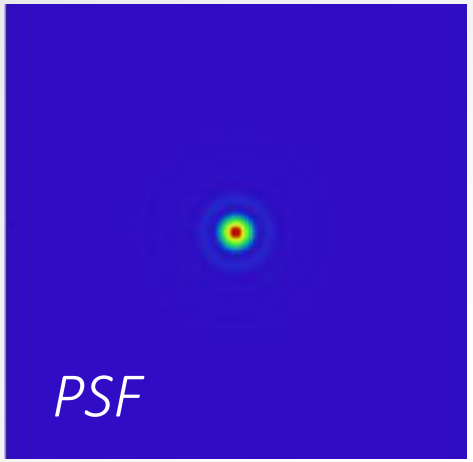
Coma



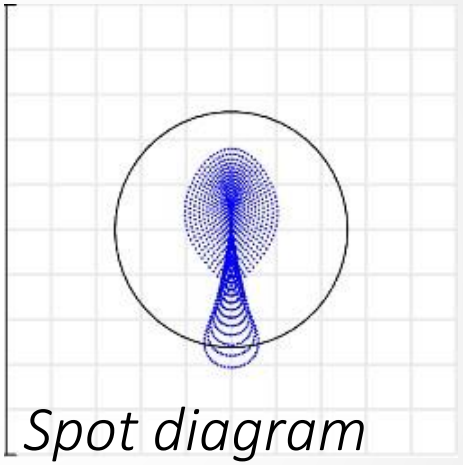
Astigmatism Through Focus

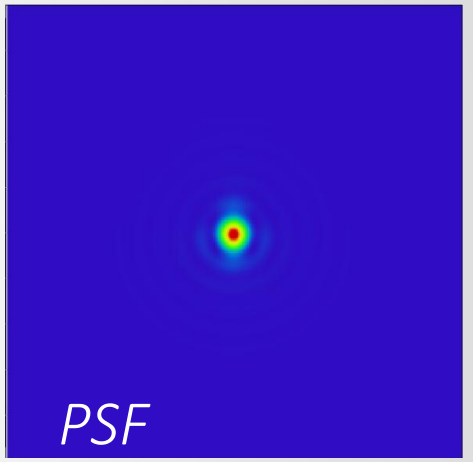
Point Spread Function (PSF)



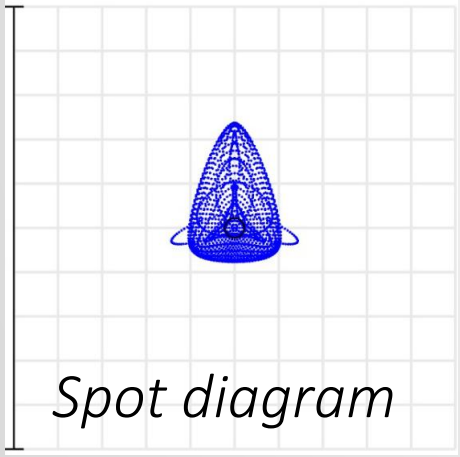


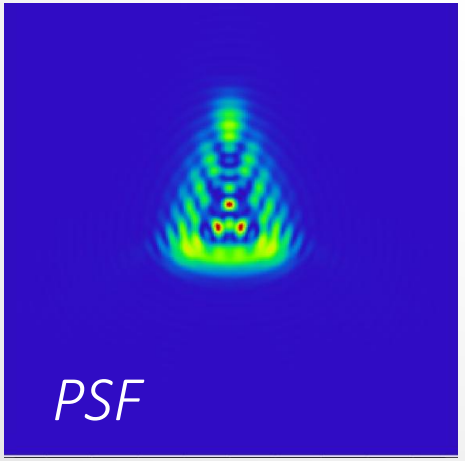
Limited by diffraction





Roughly balanced contribution





Limited by Geometric aberrations

Image Quality

Huyghens PSF vs Spot Size

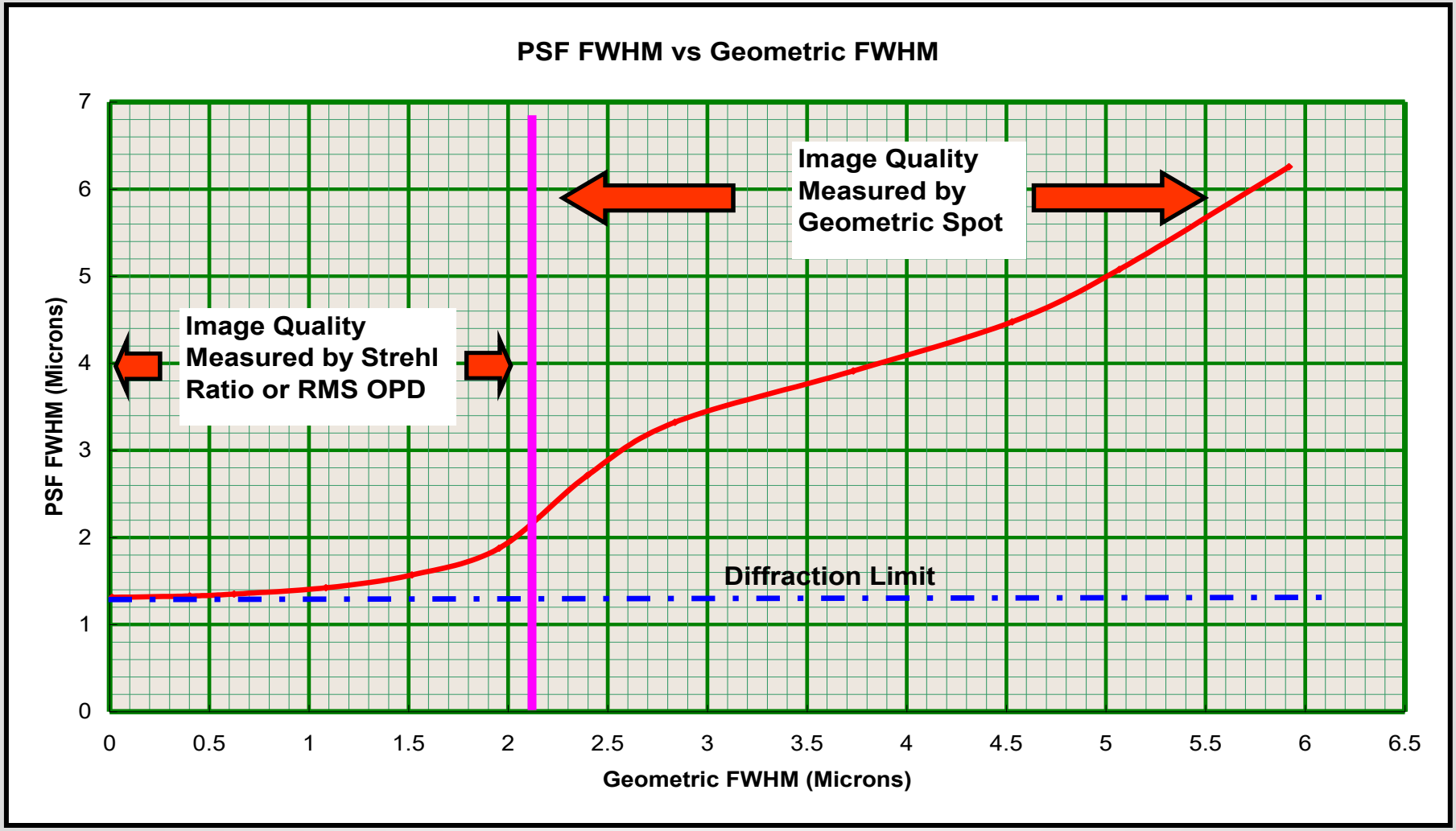


Image Quality
Measures of PSF

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- **Full width half maximum (FWHM)**
  - The displacement between the two points at which the intensity drops to half the peak
- **Encircled energy (EE)**
  - Usually 50% or 80% encircled energy
    - The radius of a circle that entirely encloses 50% (or 80%) of the PSF's total flux
- **Ensquared energy**
  - Same as encircled energy but expressed in terms of the size of a square that encloses some proportion of the PSF's flux
- **Enslitted energy**
  - Same as ensquared energy, but for one dimension only

Image Quality
Strehl Ratio

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- Strehl ratio is the ratio of the peak intensity of the aberrated PSF to the peak intensity of the unaberrated PSF
- Forms a useful arbitrary definition of 'diffraction limited' imaging
- Strehl Ratio > 0.8 is considered diffraction limited
  - Maréchal's Criterion
  - Equivalent to RMS WFE of ~ λ/14

For small WFE:

$$Strehl \approx 1 - \left(\frac{2\pi}{\lambda}\right)^2 \cdot variance$$

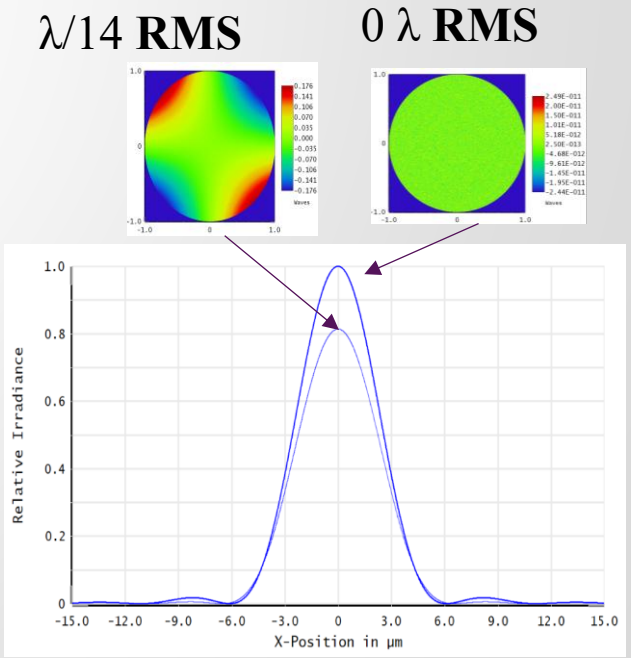
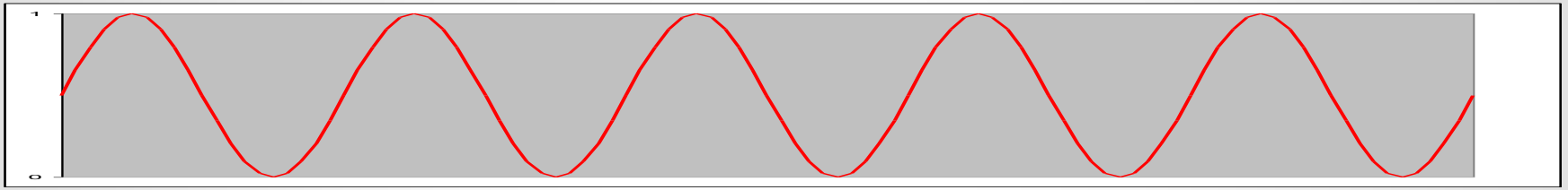


Image Quality

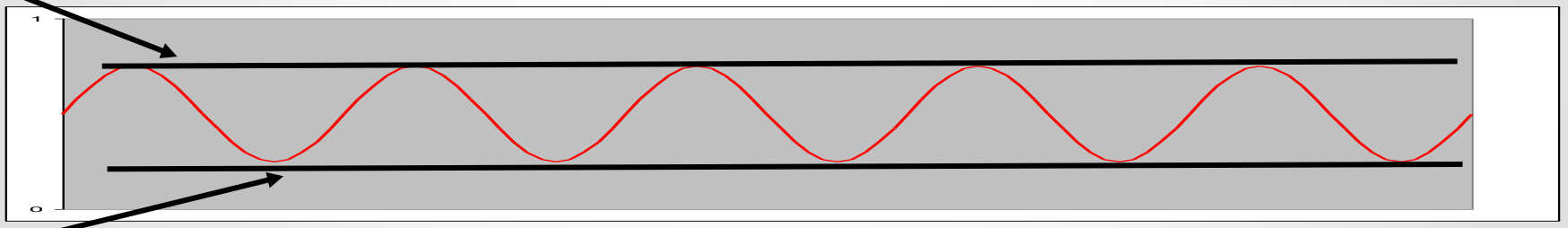
MTF

- MTF is the contrast ratio of the image of a perfect sinusoidal object of some spatial frequency f
- The MTF is given as a function of spatial frequency



Object Intensity vs Displacement

I_{max}



I_{min}

Image Intensity vs Displacement

$$MTF = \left[\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right]$$

Image Quality

MTF (2)

- Performance of an optical system is determined by the MTF at high spatial frequency (high lines per mm)

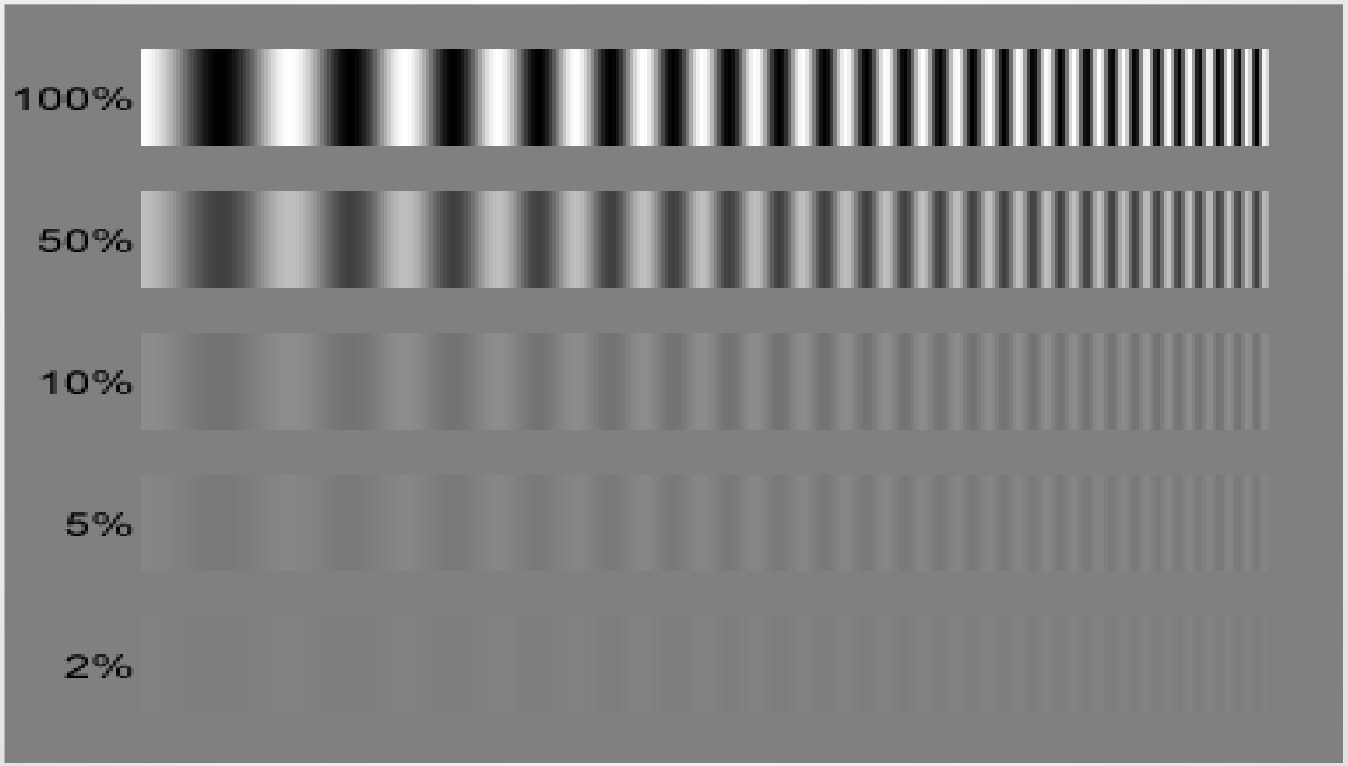
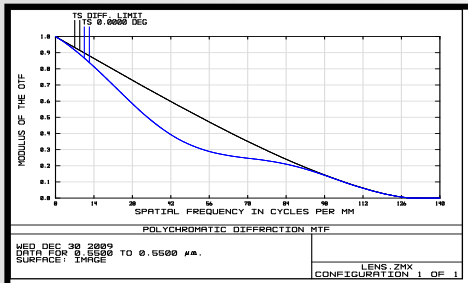


Image Quality

MTF (3)



Diffraction and Image Quality
Summary

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